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Modeling the Spread of Plant Bacterial Disease with Trivariate Stochastic Processes

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Abstract- Assessing the intensity of diseases with the spread of bacteria among plants through a trivariate stochastic model is proposed in this study. The parameters of disease spread are varying with the type and stage of the plant. This study has considered three stages of plants namely Nursery stage, Transplantation stage and completely grown/Yielding stage as three variates for getting the joint probability functions. Stochastic differential equations were formulated through trivariate point processes for deriving several statistical measures based on the proposed parameters of bacteria growth, spread and loss. Model behavior was observed through the sensitivity analysis. The statistical measures based on moments are obtained for varying values of single parameters when all the remaining parameters are unchanged.

Index Terms-Trivariate Stochastic Processes, Bacterial Diseases, Stochastic Differential Equations, Sensitivity Analysis, Statistical Measures.

I. Introduction

Bacterial diseases in plants are quite common during its different stages. Their intensity is varying and depends on several controlling and regulating factors. Certain diseases will grow at very faster rate within no time. These types of diseases will leads to heavy losses to the agriculturalists. Appropriate and timely assessment of the disease intensity will save the crop from several unwarranted situations. This activity will in turn help in boosting the healthy crop growth so as increase the expected yields of the crop . Hence there is an increasing need of these studies. Understanding the dynamics of bacterial diseases will help in designing the effective crop management, treatment protocols and intervention methods. Manual and plant pathological laboratory approaches may not be suitable for immediate assessment of certain disease intensities. Mathematical biology will help in studying the severity of the bacterial disease in terms of growth and spread. Due to many uncertainty reasons the growth and spread of these diseases are stochastic in nature. Hence, the suitable probabilistic tools can measure the essential growth and control parameters.

A Trivariate stochastic model is proposed in this study to assess the intensity and spread of the said diseases in plants by grading them as plants in nursery stage, plants in transplantation stage and plants in full grown stage. The bacterial diseases in plants are influenced by several factors and a few of them are like the age of the plant, the variety of plant, the location and environment of the plant growing, etc. The changing patterns of plant diseases in small and infinitesimal intervals of time will differentiate the plant's growth. Stochastic differential equations will help in formulating the changing intensities of the diseases among plants. Hence this study is focused on recording the variations over a period of time during (0,t) based on the changing dynamics in a time (t, t+Δt). Various steps in the study include defining assumptions and formulation of postulates based on plant pathology, developing difference equations, deriving the differential equations, obtaining probability functions through transient state of equations, deriving various statistical measures, and carrying sensitivity analysis with numerical data sets.

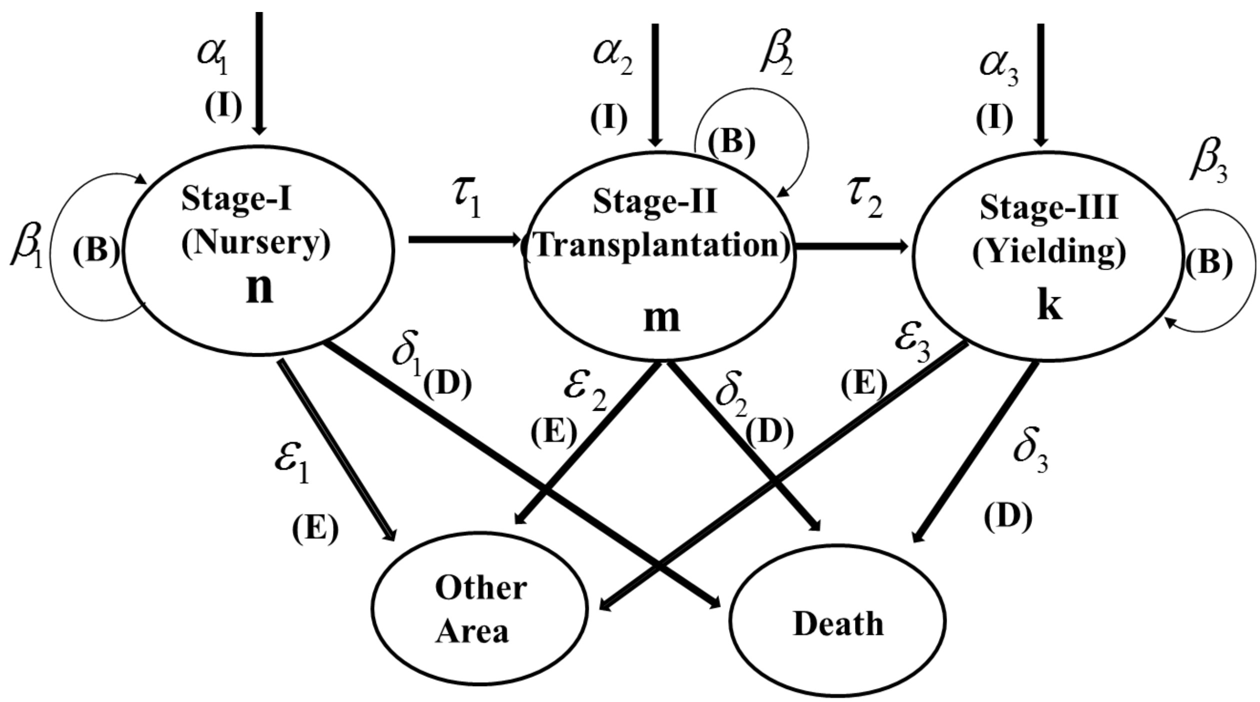
The pioneering work on mathematical theory of epidemics with deterministic model was reported in 1927 by Kermack and McKendrick. Another significant development in the contemporary period is development of chain binomial stochastic model on epidemiology by Reed and Frost (1928). Xu and Ridout (2000) have developed stochastic simulation model for studying the initial epidemic conditions and the spatial pattern of initially infected plants along with the relationships of spatio-temporal statistics and underlying biological/ physical factors. Baily (1975) contributed a significant study on epidemic modeling. Anderson and Britton (2000) have made a good overview of other important works on epidemiology modeling. A stochastic model based on a Poisson branching process for analyzing surveillance data of infectious diseases was developed by Hofmann et al. (2004). The studies on cancer growth using differential equations with Poisson postulates were contributed by Tirupathi Rao et al., (2006, 2011, 2012, 2013). Segarra et al. (2001) proposed that Infected plants lose their infectiousness and proceed into post-infectious stage at rate which is the inverse of mean infectious period.

The initial movement of pathogen will be from an infected plant. The infection of the plant may be in latent stage for much period before it becomes infected and is considered to be the inverse of mean latent period. This study has considered the philosophy of modeling behind the spread of epidemiology and cancer like diseases. Further, this study is on predicting the severity of plant based bacterial diseases with mathematical and stochastic models. The objective of the study includes on assessing the intensity of the bacteria in a unit area of plant through a trivariate stochastic model. This work is based on the invasion and expansion of the infectious diseases among plants. Exploring the indicators of the disease lethality through several arrival and departure issues of bacteria on the plant.

It is assumed that there are three stages of plant namely nursery, transplantation and full grown/yielding. Logging of bacteria on the plant at all the mentioned stages are may be due to either immigration from other plants or from the influence of some other vectors. The arrival, moving and death of bacteria at each stage of the plant may be done due to (i) new arrival of bacteria through immigration from other sources, (ii) new and internal growth of bacteria from its existing size, (iii) transferring or moving the bacteria from one stage plant to the other stage plant, (iv) moving or emigration of bacteria from each stage plant to the other areas, and (v) death of bacteria at each stage of plant, etc. The processes of birth, death and migrations will influence the dynamics of growth, loss and transition of the bacteria within same aged plant or between different aged plants.

II. TRIVARIATE STOCHASTIC MODEL FOR SPREAD OF BACTERIAL DISEASES AMONG PLANTS

This model is proposed by considering the plant pathology and other biological issues of bacterial growth, loss and migrations. The assumptions of plant diseases and bacterial growth were defined using mathematical principles. Further it is assumed that the transition of bacterial disease is purely random and influenced by complete chance factors. The study is based on the mathematical and biological postulates while constructing the model. The parameters of the model are assumed to follow Poisson processes. The following is the schematic diagram of bacterial growth, transition and loss in three stages of plants.



Schematic Diagram for Spread of Bacterial Diseases

A. Assumptions and Postulates of the model

1) Let the initial sizes of bacteria during the time (0,t) in stage-I (say nursery), stage-II (say transplantation) and stage-III (say yielding) be ‘n’, ‘m’ and ‘k’ units of bacteria respectively. Initially, here one unit denotes the number of bacteria in a square area (say mm^2).
2) Let the events be occurred in non-overlapping intervals of time and they are statistically independent.
3) Let Δt be an infinitesimal interval of time in which the phenomena of bacterial growth/transition/loss will be observed.
4) Let ‘α_1, α_2 and α_3’ be the rates of immigration of bacteria per unit time to stage-I, stage-II and stage-III plants respectively;
5) β_1, β_2 and β_3 are the rates of growth due to new arrivals (births) of bacteria per unit time in stage-I, stage-II and stage-III respectively;

6) τ_1 and τ_2 are the rates of transition of bacteria per unit time from stage-I to stage-II and from stage II to stage-III plants respectively;
7) $\varepsilon_1, \varepsilon_2$ and ε_3 are the rates of emigration of bacteria per unit time from stage-I, stage-II and stage-III plants respectively to the other areas;
8) δ_1, δ_2 and δ_3 be the rates of bacteria loss due to death per unit time in stage-I, stage-II and stage-III plants respectively.
9) Further it is assumed that the said parameters follow Poisson processes.

Basing on the above assumptions, the proposed postulates of the model are as follows:

1) The probability for arrival of bacteria to stage-I plants during Δt time through immigration from external sources is $\alpha_1 \Delta t + o(\Delta t)$;
2) The probability for arrival of bacteria to stage-I during Δt time through internal birth process provided there exits 'n' units of bacteria at time 't' is $n\beta_1 \Delta t + o(\Delta t)$;
3) The probability for transition of bacteria from stage-I to stage-II during Δt time provided there exits 'n' units of bacteria at time 't' in stage-I is $n\tau_1 \Delta t + o(\Delta t)$;
4) The probability for emigration of bacteria from stage-I to the other areas during Δt provided there exits 'n' units of bacteria at time 't' is $n\varepsilon_1 \Delta t + o(\Delta t)$;
5) The probability for death of bacteria in stage-I during Δt provided there exits 'n' units of bacteria at time 't' in stage-I is $n\delta_1 \Delta t + o(\Delta t)$;
6) The probability for arrival of bacteria to stage-II during Δt through immigration from external sources is $\alpha_2 \Delta t + o(\Delta t)$;
7) The probability for growth of bacteria to stage-II during Δt through internal birth process provided there exits 'm' units of bacteria at time 't' is $m\beta_2 \Delta t + o(\Delta t)$;
8) The probability for transition of bacteria from stage-II to stage-III during Δt provided there exits 'm' units of bacteria in stage-II at time 't' is $m\tau_2 \Delta t + o(\Delta t)$;
9) The probability for emigration of bacteria from stage-II to the other areas during Δt provided there exits 'm' units of bacteria in stage-II at time 't' is $m\varepsilon_2 \Delta t + o(\Delta t)$;
10) The probability for death of bacteria in stage-II during Δt provided there exits 'm' units of bacteria in stage-II at time 't' is $m\delta_2 \Delta t + o(\Delta t)$;
11) The probability for arrival of bacteria to stage-III during Δt through immigration from external sources is $\alpha_3 \Delta t + o(\Delta t)$;
12) The probability for growth of bacteria to stage-III during Δt through internal birth process provided there exits 'k' units of bacteria in stage-III at time 't' is $k\beta_3 \Delta t + o(\Delta t)$;
13) The probability for emigration of bacteria from stage-III to the other areas during Δt provided there exits 'k' units of bacteria in stage-III at time 't' is $k\varepsilon_3 \Delta t + o(\Delta t)$;
14) The probability for death of bacteria in stage-III during Δt provided there exits 'k' units of bacteria in stage-III at time 't' is $k\delta_3 \Delta t + o(\Delta t)$;
15) The probability for no arrival of bacteria to stage-I, stage-II and stage-III; no birth of bacteria in stage-I, stage-II and stage-III; no transition of bacteria from stage-I to stage-II and from stage-II to stage-III; no emigration of bacteria from stage-I, stage-II and stage-III; to the other area and no death of bacteria in stage-I, stage-II and stage-III; during an infinitesimal interval of time Δt is

$$1-\{\alpha_1 + n(\beta_1+\tau_1)+\alpha_2+m(\beta_2+\tau_2)+\alpha_3+k\beta_3+n(\varepsilon_1+\delta_1)+m(\varepsilon_2+\delta_2)+k(\varepsilon_3+\delta_3)\}.\Delta t+o(\Delta t)$$

16) The probability for occurrence of other than the above events during an infinitesimal interval of time Δt is $O(\Delta t)^2$.

B. Differential Difference Equations of the Model

Let $P_{n,m,k}(t)$ be the joint probability of existence of 'n', 'm' and 'k' units of bacteria in stage-I, stage-II and stage-III respectively per unit time't'.

$$\begin{aligned}
P_{n,m,k}(t+\Delta t) = P_{n,m,k}(t)[1-\{&\alpha_1 + n(\beta_1+\tau_1)+\alpha_2+m(\beta_2+\tau_2)+\alpha_3+k\beta_3 \\
&+ n(\varepsilon_1+\delta_1)+m(\varepsilon_2+\delta_2)+k(\varepsilon_3+\delta_3)\}.\Delta t + O(\Delta t)] \\
&+ P_{n,m,k-1}(t)\left[\{\alpha_3+(k-1)\beta_3\}\Delta t+O(\Delta t)\right]+P_{n,m,k+1}(t)\left[\{(k+1)(\varepsilon_3+\delta_3)\}\Delta t+O(\Delta t)\right] \\
&+ P_{n,m-1,k}(t)\left[\{\alpha_2+(m-1)\beta_2\}\Delta t+O(\Delta t)\right]+P_{n,m+1,k}(t)[\{(m+1)(\varepsilon_2+\delta_2)\}.\Delta t+O(\Delta t)] \\
&+ P_{n,m+1,k-1}(t)[\{(m+1)\tau_2\}.\Delta t+O(\Delta t)]+P_{n-1,m,k}(t)[\{\alpha_1+(n-1)\beta_1\}\Delta t+O(\Delta t)] \\
&+ P_{n+1,m,k}(t)[\{(n+1)(\varepsilon_1+\delta_1)\}.\Delta t+O(\Delta t)]+P_{n+1,m-1,k}(t)[\{(n+1)\tau_1\}.\Delta t+O(\Delta t)] \\
&+ P_{n\pm i,m\pm i,k\pm i}(t)[O(\Delta t)^2] \quad for\ i\geq 2
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}P_{n,m,k}(t) = &\begin{bmatrix} -\{\alpha_1+n(\beta_1+\tau_1)+\alpha_2+m(\beta_2+\tau_2)+\alpha_3+k\beta_3 \\ \qquad + n(\varepsilon_1+\delta_1)+m(\varepsilon_2+\delta_2)+k(\varepsilon_3+\delta_3)\}.P_{n,m,k}(t)\end{bmatrix} \\
&+\left[\{\alpha_3+(k-1)\beta_3\}.P_{n,m,k-1}(t)\right]+\left[\{(k+1)(\varepsilon_3+\delta_3)\}.P_{n,m,k+1}(t)\right] \\
&+\left[\{\alpha_2+(m-1)\beta_2\}.P_{n,m-1,k}(t)\right]+[\{(m+1)(\varepsilon_2+\delta_2)\}.P_{n,m+1,k}(t)] \\
&+[\{(m+1)\tau_2\}.P_{n,m+1,k-1}(t)]+[\{\alpha_1+(n-1)\beta_1\}P_{n-1,m,k}(t)] \\
&+[\{(n+1)(\varepsilon_1+\delta_1)\}.P_{n+1,m,k}(t)]+[\{(n+1)\tau_1\}.P_{n+1,m-1,k}(t)] \quad for\ n,m,k\geq 1
\end{aligned}$$

Other differential equations for n, m, k = 0,1 are

$$\begin{aligned}
\frac{d}{dt}P_{0,0,1}(t) = &-\left(\alpha_1+\alpha_2+\alpha_3+\beta_3+\varepsilon_3+\delta_3\right)P_{0,0,1}(t)+\left(\alpha_3\right)P_{0,0,0}(t)+2\left(\varepsilon_3+\delta_3\right)P_{0,0,2}(t)+ \\
&\left(\varepsilon_2+\delta_2\right)P_{0,1,1}(t)+\left(\tau_2\right)P_{0,1,0}(t)+\left(\varepsilon_1+\delta_1\right)P_{1,0,1}(t)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}P_{0,1,0}(t) = &-\left(\alpha_1+\alpha_2+\alpha_3+\beta_2+\tau_2+\varepsilon_2+\delta_2\right)P_{0,1,0}(t)+\left(\varepsilon_3+\delta_3\right)P_{0,1,1}(t)+\left(\alpha_2\right)P_{0,0,0}(t)+ \\
&2\left(\varepsilon_2+\delta_2\right)P_{0,2,0}(t)+\left(\varepsilon_1+\delta_1\right)P_{1,1,0}(t)+\left(\tau_1\right)P_{1,0,0}(t)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}P_{1,0,0}(t) = &-\left(\alpha_1+\alpha_2+\alpha_3+\beta_1+\tau_1+\varepsilon_1+\delta_1\right)P_{1,0,0}(t)+\left(\varepsilon_3+\delta_3\right)P_{1,0,1}(t)+\left(\varepsilon_2+\delta_2\right)P_{1,1,0}(t)+ \\
&\left(\alpha_1\right)P_{0,0,0}(t)+2\left(\varepsilon_1+\delta_1\right)P_{2,0,0}(t)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}P_{1,1,0}(t) = &-\left(\alpha_1+\alpha_2+\alpha_3+\beta_1+\tau_1+\beta_2+\tau_2+\varepsilon_1+\delta_1+\varepsilon_2+\delta_2\right)P_{1,1,0}(t)+\left(\varepsilon_3+\delta_3\right)P_{1,1,1}(t)+ \\
&\left(\alpha_2\right)P_{1,0,0}(t)+2\left(\varepsilon_2+\delta_2\right)P_{1,2,0}(t)+\left(\alpha_1\right)P_{0,1,0}(t)+2\left(\varepsilon_1+\delta_1\right)P_{2,1,0}(t)+2\left(\tau_1\right)P_{2,0,0}(t)
\end{aligned}$$

$$\frac{d}{dt}P_{1,0,1}(t)=-\left(\alpha_1+\alpha_2+\alpha_3+\beta_1+\tau_1+\beta_3+\varepsilon_1+\delta_1+\varepsilon_3+\delta_3\right)P_{1,0,1}(t)+\left(\alpha_3\right)P_{1,0,0}(t)+ 2\left(\varepsilon_3+\delta_3\right)P_{1,0,2}(t)+\left(\varepsilon_2+\delta_2\right)P_{1,1,1}(t)+\left(\tau_2\right)P_{1,1,0}(t)+\left(\alpha_1\right)P_{0,0,1}(t)+2\left(\varepsilon_1+\delta_1\right)P_{2,0,1}(t)$$

$$\frac{d}{dt}P_{0,1,1}(t)=-\left(\alpha_1+\alpha_2+\alpha_3+\beta_2+\tau_2+\beta_3+\varepsilon_2+\delta_2+\varepsilon_3+\delta_3\right)P_{0,1,1}(t)+\left(\alpha_3\right)P_{0,1,0}(t)+ 2\left(\varepsilon_3+\delta_3\right)P_{0,1,2}(t)+\left(\alpha_2\right)P_{0,0,1}(t)+2\left(\varepsilon_2+\delta_2\right)P_{0,2,1}(t)+2\left(\tau_2\right)P_{0,2,0}(t)+ \left(\varepsilon_1+\delta_1\right)P_{1,1,1}(t)+\left(\tau_1\right)P_{1,0,1}(t)$$

The initial conditions are $P_{n,m,k}(t)=0;$ for n<N$_0$; m<M$_0$; k<K$_0$; for t=0 and $P_{n,m,k}(t)=1;$ for n=N$_0$; m=M$_0$; k=K$_0$;

Using the boundary conditions and differential-difference equations, the probability generating function (p.g.f.) is, $p\left(x,y,z;t\right)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}x^n y^m z^k p_{n,m,k}(t)$.

Differentiating on both sides, we get

$$\frac{d}{dt}p\left(x,y,z;t\right)=\frac{d}{dt}\left(\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}x^n y^m z^n p_{n,m,k}(t)\right)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}x^n y^m z^n.\frac{d}{dt}\left(p_{n,m,k}(t)\right)$$

The above equation can be solved by multiplying the differential-difference equation with $x^n y^m$ on both sides and summing over n, m from 0 to ∞ and using the approaches of cumulant generating function (c.g.f.).

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}x^n y^m z^n\frac{d}{dt}\left(p_{n,m,k}(t)\right)=$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left[\begin{array}{l}-\{\alpha_1+n(\beta_1+\tau_1)+\alpha_2+m(\beta_2+\tau_2)+\alpha_3+k\beta_3\\ \quad +n(\varepsilon_1+\delta_1)+m(\varepsilon_2+\delta_2)+k(\varepsilon_3+\delta_3)\}x^n y^m z^n.P_{n,m,k}(t)+\\ \left[\left\{\alpha_3+(k-1)\beta_3\right\}x^n y^m z^n.P_{n,m,k-1}(t)\right]+\left[\left\{(k+1)(\varepsilon_3+\delta_3)\right\}x^n y^m z^n.P_{n,m,k+1}(t)\right]\\ +\left[\left\{\alpha_2+(m-1)\beta_2\right\}x^n y^m z^n.P_{n,m-1,k}(t)\right]+[\left\{(m+1)(\varepsilon_2+\delta_2)\right\}x^n y^m z^n.P_{n,m+1,k}(t)]\\ +[\left\{(m+1)\tau_2\right\}x^n y^m z^n.P_{n,m+1,k-1}(t)]+[\left\{\alpha_1+(n-1)\beta_1\right\}x^n y^m z^n.P_{n-1,m,k}(t)]+\\ [\left\{(n+1)(\varepsilon_1+\delta_1)\right\}x^n y^m z^n.P_{n+1,m,k}(t)]+[\left\{(n+1)\tau_1\right\}x^n y^m z^n.P_{n+1,m-1,k}(t)]\end{array}\right]$$

The statistical measures after solving the above differential equations using initial conditions and joint cumulant generating function are:

Expected number of bacterial units in stage-I at time ‘t’ is

$$m_{1,0,0}(t)\ =\ N_o.e^{At} \qquad (2.2.1)$$

Expected number of bacterial units in stage-II at time ‘t’ is

$$m_{0,1,0}(t)\ =\ \frac{\tau_1 N_o.e^{At}}{(A-B)}+M_o.e^{Bt} \qquad (2.2.2)$$

Expected number of bacterial units in stage-III at time ‘t’ is

$$m_{0,0,1}(t) = \frac{\tau_1 \tau_2 N_o.e^{At}}{(A-B)(A-C)} + \frac{\tau_2 M_o.e^{Bt}}{(B-C)} + K_o.e^{Ct} \quad (2.2.3)$$

Variance of number of units of bacteria in stage-I at time 't' is

$$m_{2,0,0}(t) = \frac{-\left(2\alpha_1 + \beta_1 + \tau_1\right) N_o.e^{At}}{A} + C_o.e^{2At} \quad (2.2.4)$$

Variance of number of units of bacteria in stage-II at time 't' is

$$\begin{aligned} m_{0,2,0}(t) = & \frac{\tau_1 N_o.e^{At}}{(A-2B)} + \frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)} + \frac{\left(-JM_o.e^{Bt}\right)}{(B)} + \frac{\left\{-2\tau_1(\alpha_2 - \tau_1)N_o.e^{At}\right\}}{(A-2B)(B)} \\ & + \frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)} + \frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)} + \frac{\left\{2\tau_1^2(2\alpha_1 + \beta_1 + \tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)} \\ & + \frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2} + \frac{D_o.e^{(A+B)t}}{(A-B)} + E_o.e^{2Bt} \end{aligned} \quad (2.2.5)$$

Variance of number of units of bacteria in stage-III at time 't' is

$$
\begin{aligned}
m_{0,0,2}(t) = & \frac{\tau_1\tau_2 N_o.e^{At}}{(A-B)(A-2C)} + \frac{\left(\tau_2 M_o.e^{Bt}\right)}{(B-2C)} + \frac{\left(D\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(A-C)(A-2C)} + \frac{\left(D\tau_2 M_o.e^{Bt}\right)}{(B-C)(B-2C)} \\
& + \frac{\left(-DK_o.e^{Ct}\right)}{(C)} + \frac{\left\{2\tau_1\tau_2(\alpha_3-\tau_2)N_o.e^{At}\right\}}{(A-B)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2(\alpha_3-\tau_2)M_o.e^{Bt}\right\}}{C(B-2C)} \\
& + \frac{2\alpha_2\tau_1\tau_2^2 N_o.e^{At}}{(A-B)(A-C)(A-B-C)(A-2C)} + \frac{\left\{-2\tau_2^2\alpha_2 M_o.e^{Bt}\right\}}{(B-C)C(B-2C)} + \frac{\left(2\alpha_2\tau_2 K_o.e^{Ct}\right)}{(BC)} \\
& + \frac{-2\tau_1\tau_2\alpha_3 N_o.e^{At}}{C(A-B-C)(A-2C)} + \frac{\left\{-2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)(A-C)C(A-B-C)(A-2C)} \\
& + \frac{\left\{-2\tau_1\tau_2^2\alpha_1 M_o.e^{Bt}\right\}}{(B-C)(B-A-C)C(B-2C)} + \frac{\left\{-2\tau_1\tau_2\alpha_1 K_o.e^{Ct}\right\}}{(ABC)} + \frac{\left\{2\tau_1\tau_2^2(\alpha_2-\tau_2)N_o.e^{At}\right\}}{BC(A-B-C)(A-2C)} \\
& + \frac{\left\{2\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)BC(A-B-C)(A-2C)} + \frac{\left\{2\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{AC(B-A-C)(B-2C)} + \frac{-2\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{ABC(A-B-C)(A-2C)} \\
& + \frac{\tau_1^2\tau_2^2 C_o.e^{2At}}{(A-B)(A-C)^2(2A-B-C)} + \frac{\left\{2\tau_1\tau_2^2 D_o.e^{(A+B)t}\right\}}{(B-C)(A-C)(A+B-2C)} + \frac{\left(2\tau_1\tau_2 F_o.e^{(A+C)t}\right)}{(A-B)(A-C)} \\
& + \frac{2\tau_1\tau_2^2 N_o.e^{At}}{(A-2B)(A-B-C)(A-2C)} + \frac{\left(J2\tau_1\tau_2^2 N_o.e^{Bt}\right)}{(A-B)BC(B-2C)} + \frac{\left(J2\tau_2^2 M_o.e^{Bt}\right)}{BC(B-2C)} \\
& + \frac{\left\{-4\tau_1\tau_2^2(\alpha_2-\tau_1)N_o.e^{At}\right\}}{B(A-2B)(A-2C)(A-B-C)} + \frac{\left\{-4\alpha_1\tau_1^2\tau_2^2 N_o.e^{At}\right\}}{(A-B)B(A-2B)(A-2C)(A-B-C)} \\
& + \frac{\left\{-4\alpha_1\tau_1\tau_2^2 M_o.e^{Bt}\right\}}{ABC(B-2C)} + \frac{4\tau_1^2\tau_2^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{AB(A-2B)(A-B-C)(A-2C)} + \frac{\left(\tau_1^2\tau_2^2 C_o.e^{2At}\right)}{(A-C)(A-B)^2(2A-B-C)} \\
& + \frac{2\tau_2^2 D_o.e^{(A+B)t}}{(A-B)(A-C)(A+B-2C)} + \frac{\left(\tau_2^2 E_o.e^{(2B)t}\right)}{(B-C)^2} + \frac{2\tau_2 G_o.e^{(B+C)t}}{(B-C)} + H_o.e^{(2C)t}
\end{aligned} \qquad (2.2.6)
$$

Covariance of number of units of bacteria of stage-I and stage-II at time ‘t’ is

$$
\begin{aligned}
m_{1,1,0}(t) = & \frac{\left\{-(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(B)} + \frac{\left(-\alpha_1\tau_1 N_o.e^{At}\right)}{(A-B)(B)} + \frac{\left(-\alpha_1 M_o.e^{Bt}\right)}{(A)} + \frac{\left\{\tau_1(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A)(B)} \\
& + \frac{\left(\tau_1 C_o.e^{2At}\right)}{(A-B)} + D_o.e^{(A+B)t}
\end{aligned} \qquad (2.2.7)
$$

Covariance of number of units of bacteria of stage-I and stage-III at time ‘t’ is

$$m_{1,0,1}(t)=\frac{\left(-\alpha_3 N_o.e^{At}\right)}{(C)}+\frac{\left(-\alpha_1\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(A-C)(C)}+\frac{\left(\alpha_1\tau_2 M_o.e^{Bt}\right)}{(B-C)(B-A-C)}+\frac{\left(-\alpha_1 K_o.e^{Ct}\right)}{(A)}$$
$$+\frac{\left\{\tau_2(\alpha_2-\tau_2)N_o.e^{At}\right\}}{(B)(C)}+\frac{\left(\alpha_1\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(B)(C)}+\frac{\left(-\alpha_1\tau_2 M_o.e^{Bt}\right)}{(A)(B-A-C)} \quad (2.2.8)$$
$$+\frac{\left\{-\tau_1\tau_2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A)(B)}+\frac{\left(\tau_1\tau_2 C_o.e^{2At}\right)}{(A-B)(A-C)}+\frac{\left(\tau_2 D_o.e^{(A+B)t}\right)}{(B-C)}+F_o.e^{(A+C)t}$$

Covariance of number of units of bacteria of stage-II and stage-III at time 't' is

$$m_{0,1,1}(t)=\frac{(\alpha_3-\tau_2)\tau_1 N_o.e^{At}}{(A-B)(A-B-C)}+\frac{\left(-\left(\alpha_3-\tau_2\right)M_o.e^{Bt}\right)}{(C)}+\frac{\left(\alpha_2\tau_1\tau_2 N_o.e^{At}\right)}{(A-B)(A-B-C)(A-C)}+\frac{\left(-\alpha_2\tau_2 M_o.e^{Bt}\right)}{(C)(B-C)}$$
$$+\frac{\left(-\alpha_2 K_o.e^{Ct}\right)}{(B)}+\frac{\left(-\alpha_3\tau_1 N_o.e^{At}\right)}{(C)(A-B-C)}+\frac{\left(-\alpha_1\tau_1^2\tau_2 N_o.e^{At}\right)}{(A-B)(A-C)C(A-B-C)}+\frac{\left(-\alpha_1\tau_1\tau_2 M_o.e^{Bt}\right)}{(B-C)(B-A-C)C}+\frac{\left(\alpha_1\tau_1 K_o.e^{Ct}\right)}{AB}$$
$$+\frac{\left\{\tau_1\tau_2(\alpha_2-\tau_2)N_o.e^{At}\right\}}{(BC)(A-B-C)}+\frac{\left(\alpha_1\tau_1^2\tau_2 N_o.e^{At}\right)}{(A-B)(BC)C(A-B-C)}+\frac{\left(\alpha_1\tau_1\tau_2 M_o.e^{Bt}\right)}{(AC)(B-A-C)}+\frac{\left\{-\tau_1^2\tau_2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{ABC(A-B-C)}$$
$$+\frac{\left(\tau_1^2\tau_2 C_o.e^{2At}\right)}{(A-B)(A-C)(2A-B-C)}+\frac{\left(\tau_1\tau_2 D_o.e^{(A+B)t}\right)}{(B-C)(A-C)}+\frac{\left(\tau_1 F_o.e^{(A+C)t}\right)}{(A-B)}+\frac{\left(\tau_1\tau_2 N_o.e^{At}\right)}{(A-2B)(A-B-C)}+\frac{\left(J\tau_1\tau_2 N_o.e^{Bt}\right)}{(A-B)BC}$$
$$+\frac{\left(J\tau_2 M_o.e^{Bt}\right)}{BC}+\frac{\left\{-2\tau_1\tau_2(\alpha_2-\tau_1)N_o.e^{At}\right\}}{B(A-2B)(A-B-C)}+\frac{-2\alpha_1\tau_1^2\tau_2 N_o.e^{At}}{(A-B)B(A-2B)(A-B-C)}+\frac{-2\alpha_1\tau_1^2\tau_2 M_o.e^{Bt}}{ABC}+$$
$$\frac{2\tau_1^2\tau_2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}}{AB(A-2B)(A-B-C)}+\frac{\tau_1^2\tau_2 C_o.e^{2At}}{(A-B)^2(2A-B-C)}+\frac{\left(\tau_2 D_o.e^{(A+B)t}\right)}{(A-B)(A-C)}+\frac{\left(\tau_2 E_o.e^{(2B)t}\right)}{(B-C)}+G_o.e^{(B+C)t}$$
$$(2.2.9)$$

Where

$A=\left[\beta_1-(\tau_1+\varepsilon_1+\delta_1)\right]; B=\left[\beta_2-(\tau_2+\varepsilon_2+\delta_2)\right]; C=\left[\beta_3-(\varepsilon_3+\delta_3)\right]; D=\left[2\alpha_3+\beta_3+\varepsilon_3+\delta_3\right];$
$J=\left[2\alpha_2+\beta_2+\tau_2+\varepsilon_2+\delta_2\right]$; N_o, M_o, K_o are initial values and C_o, D_o, E_o, F_o, G_o, H_o are constants which can be evaluated.

III NUMERICAL ILLUSTRATION

In order to verify model behavior, a hypothetical numerical data set is obtained for various statistical measures from equations from 2.2.1 to 2.2.9, such as average number of bacteria units on first stage, second stage and third stage; variances of bacterial units in first, second and third stages; and covariance between the number of bacterial units in first and second stages, second and third stages, first and third stages. While computing the values of $m_{100}(t)$, $m_{010}(t)$, $m_{001}(t)$, $m_{200}(t)$, $m_{020}(t)$, $m_{002}(t)$, $m_{110}(t)$, $m_{101}(t)$ and $m_{011}(t)$ with MATHCAD, it is considered for changing values one parameter and for the fixed values of the remaining parameters among α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; N_0; M_0; K_0 and t.

Table-3.1: The values of $m_{100}(t)$, $m_{010}(t)$, $m_{001}(t)$, $m_{200}(t)$, $m_{020}(t)$, $m_{002}(t)$, $m_{110}(t)$, $m_{101}(t)$ and $m_{011}(t)$ for changing and fixed values of α_1; β_1; τ_1; α_2; β_2; τ_2; α_3; β_3; ε_1; δ_1; ε_2; δ_2; ε_3; δ_3; N_0; M_0; K_0 and t

α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3	No	Mo	Ko	t	m_{100}	m_{010}	m_{001}	m_{200}	m_{020}	m_{002}	m_{110}	m_{101}	m_{011}
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
12	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.33E+04	2.26E+03	1.37E+07	5.49E+03	1.56E+05	6.80E+04
13	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.40E+04	2.37E+03	1.44E+07	5.77E+03	1.63E+05	7.09E+04
14	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.47E+04	2.48E+03	1.50E+07	6.05E+03	1.70E+05	7.37E+04
15	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.54E+04	2.59E+03	1.57E+07	6.32E+03	1.77E+05	7.65E+04
16	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.61E+04	2.70E+03	1.64E+07	6.60E+03	1.84E+05	7.93E+04
11	**9**	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	**9.5**	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	530.233	212.172	4.15E+03	2.13E+04	3.37E+03	7.53E+06	8.47E+03	2.06E+05	8.54E+04
11	**10**	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	733.859	282.332	3.89E+03	3.62E+04	5.32E+03	4.53E+06	1.39E+04	2.88E+05	1.14E+05
11	**10.5**	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	1.02E+03	376.258	3.48E+03	6.21E+04	8.49E+03	2.40E+06	2.29E+04	4.08E+05	1.53E+05
11	**11**	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	1.41E+03	502.128	2.85E+03	1.07E+05	1.37E+04	5.31E+05	3.83E+04	5.81E+05	2.09E+05
11	9	**3**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	1.41E+03	301.308	3.60E+03	9.61E+04	4.68E+03	7.95E+04	2.14E+04	3.69E+05	8.24E+04
11	9	**3.5**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	1.02E+03	263.404	3.85E+03	5.72E+04	4.01E+03	1.09E+06	1.52E+04	2.97E+05	8.06E+04
11	9	**4**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	733.859	225.881	4.05E+03	3.43E+04	3.33E+03	2.78E+06	1.07E+04	2.37E+05	7.65E+04
11	9	**4.5**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	530.233	190.963	4.21E+03	2.07E+04	2.70E+03	5.95E+06	7.49E+03	1.88E+05	7.11E+04
11	9	**5**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	**5.5**	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	276.806	132.464	4.43E+03	7.76E+03	1.69E+03	3.61E+07	3.62E+03	1.17E+05	5.93E+04
11	9	3	**14**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	**15**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.17E+03	1.31E+07	5.25E+03	1.53E+05	6.73E+04
11	9	3	**16**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.18E+03	1.32E+07	5.28E+03	1.57E+05	6.95E+04
11	9	3	**17**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.20E+03	1.33E+07	5.32E+03	1.61E+05	7.16E+04
11	9	3	**18**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.21E+03	1.34E+07	5.35E+03	1.65E+05	7.37E+04
11	9	3	**19**	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.23E+03	1.35E+07	5.39E+03	1.70E+05	7.58E+04
11	9	3	14	**8**	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	**9**	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	174.29	4.30E+03	1.26E+04	2.52E+03	1.73E+07	5.65E+03	1.48E+05	7.07E+04
11	9	3	14	**10**	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	191.842	4.25E+03	1.26E+04	3.00E+03	2.37E+07	6.17E+03	1.46E+05	7.71E+04
11	9	3	14	**11**	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	213.39	4.20E+03	1.26E+04	3.62E+03	3.37E+07	6.79E+03	1.44E+05	8.47E+04
11	9	3	14	**12**	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	240.499	4.13E+03	1.26E+04	4.47E+03	4.96E+07	7.54E+03	1.39E+05	9.45E+04
11	9	3	14	8	**15**	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04

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α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3	No	Mo	Ko	t	m_{100}	m_{010}	m_{001}	m_{200}	m_{020}	m_{002}	m_{110}	m_{101}	m_{011}
11	9	3	14	8	16	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	147.39	4.34E+03	1.26E+04	1.86E+03	1.09E+07	4.84E+03	1.45E+05	5.89E+04
11	9	3	14	8	17	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	136.846	4.34E+03	1.26E+04	1.62E+03	9.13E+06	4.51E+03	1.41E+05	5.34E+04
11	9	3	14	8	18	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	127.714	4.35E+03	1.26E+04	1.42E+03	7.66E+06	4.23E+03	1.37E+05	4.86E+04
11	9	3	14	8	19	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	119.727	4.35E+03	1.26E+04	1.26E+03	6.40E+06	3.98E+03	1.33E+05	4.44E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	12	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.31E+07	5.21E+03	1.54E+05	6.73E+04
11	9	3	14	8	15	13	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.32E+07	5.21E+03	1.59E+05	6.95E+04
11	9	3	14	8	15	14	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.33E+07	5.21E+03	1.64E+05	7.16E+04
11	9	3	14	8	15	15	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.35E+07	5.21E+03	1.69E+05	7.37E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	10	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	8.72E+03	1.26E+04	2.15E+03	4.52E+07	5.21E+03	2.40E+05	1.07E+05
11	9	3	14	8	15	11	11	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	1.71E+04	1.26E+04	2.15E+03	1.56E+08	5.21E+03	3.98E+05	1.80E+05
11	9	3	14	8	15	11	12	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	3.30E+04	1.26E+04	2.15E+03	5.33E+08	5.21E+03	6.75E+05	3.10E+05
11	9	3	14	8	15	11	13	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	6.35E+04	1.26E+04	2.15E+03	1.82E+09	5.21E+03	1.17E+06	5.41E+05
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	1.02E+03	376.258	3.48E+03	5.97E+04	8.18E+03	2.07E+06	2.21E+04	3.92E+05	1.48E+05
11	9	3	14	8	15	11	9	1	0.25	3	1	1	1	200	100	50	0.65	733.859	282.332	3.89E+03	3.53E+04	5.19E+03	4.28E+06	1.35E+04	2.81E+05	1.11E+05
11	9	3	14	8	15	11	9	1.5	0.25	3	1	1	1	200	100	50	0.65	530.233	212.172	4.15E+03	2.10E+04	3.33E+03	7.37E+06	8.36E+03	2.03E+05	8.44E+04
11	9	3	14	8	15	11	9	2	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	2.5	0.25	3	1	1	1	200	100	50	0.65	276.806	120.429	4.45E+03	7.65E+03	1.40E+03	2.92E+07	3.28E+03	1.10E+05	5.12E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	623.792	244.703	4.03E+03	2.72E+04	4.15E+03	5.65E+06	1.06E+04	2.39E+05	9.66E+04
11	9	3	14	8	15	11	9	0.5	0.5	3	1	1	1	200	100	50	0.65	530.233	212.172	4.15E+03	2.10E+04	3.33E+03	7.37E+06	8.36E+03	2.03E+05	8.44E+04
11	9	3	14	8	15	11	9	0.5	0.75	3	1	1	1	200	100	50	0.65	450.707	184.041	4.25E+03	1.63E+04	2.67E+03	9.66E+06	6.59E+03	1.73E+05	7.41E+04
11	9	3	14	8	15	11	9	0.5	1	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	1.25	3	1	1	1	200	100	50	0.65	325.648	138.652	4.40E+03	9.82E+03	1.74E+03	1.84E+07	4.13E+03	1.27E+05	5.77E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	4	1	1	1	200	100	50	0.65	383.108	147.39	4.36E+03	1.26E+04	1.86E+03	1.01E+07	4.84E+03	1.48E+05	6.04E+04
11	9	3	14	8	15	11	9	0.5	0.25	5	1	1	1	200	100	50	0.65	383.108	136.846	4.39E+03	1.26E+04	1.62E+03	8.14E+06	4.51E+03	1.48E+05	5.61E+04
11	9	3	14	8	15	11	9	0.5	0.25	6	1	1	1	200	100	50	0.65	383.108	127.714	4.41E+03	1.26E+04	1.42E+03	6.72E+06	4.23E+03	1.48E+05	5.23E+04
11	9	3	14	8	15	11	9	0.5	0.25	7	1	1	1	200	100	50	0.65	383.108	119.727	4.43E+03	1.26E+04	1.26E+03	5.69E+06	3.98E+03	1.48E+05	4.89E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	2	1	1	200	100	50	0.65	383.108	147.39	4.36E+03	1.26E+04	1.86E+03	1.01E+07	4.84E+03	1.48E+05	6.04E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	3	1	1	200	100	50	0.65	383.108	136.846	4.39E+03	1.26E+04	1.62E+03	8.14E+06	4.51E+03	1.48E+05	5.61E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	4	1	1	200	100	50	0.65	383.108	127.714	4.41E+03	1.26E+04	1.42E+03	6.72E+06	4.23E+03	1.48E+05	5.23E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	5	1	1	200	100	50	0.65	383.108	119.727	4.43E+03	1.26E+04	1.26E+03	5.69E+06	3.98E+03	1.48E+05	4.89E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1.5	1	200	100	50	0.65	383.108	159.707	2.98E+03	1.26E+04	2.15E+03	6.81E+06	5.21E+03	1.18E+05	5.12E+04

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α_1	β_1	τ_1	α_2	β_2	τ_2	α_3	β_3	ε_1	δ_1	ε_2	δ_2	ε_3	δ_3	No	Mo	Ko	t	m_{100}	m_{010}	m_{001}	m_{200}	m_{020}	m_{002}	m_{110}	m_{101}	m_{011}
11	9	3	14	8	15	11	9	0.5	0.25	3	1	2	1	200	100	50	0.65	383.108	159.707	1.99E+03	1.26E+04	2.15E+03	3.51E+06	5.21E+03	9.37E+04	4.04E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	2.5	1	200	100	50	0.65	383.108	159.707	1.25E+03	1.26E+04	2.15E+03	1.75E+06	5.21E+03	7.49E+04	3.20E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	3	1	200	100	50	0.65	383.108	159.707	690.837	1.26E+04	2.15E+03	8.10E+05	5.21E+03	6.00E+04	2.53E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1.5	200	100	50	0.65	383.108	159.707	2.98E+03	1.26E+04	2.15E+03	6.81E+06	5.21E+03	1.18E+05	5.12E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	2	200	100	50	0.65	383.108	159.707	1.99E+03	1.26E+04	2.15E+03	3.51E+06	5.21E+03	9.37E+04	4.04E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	2.5	200	100	50	0.65	383.108	159.707	1.25E+03	1.26E+04	2.15E+03	1.75E+06	5.21E+03	7.49E+04	3.20E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	3	200	100	50	0.65	383.108	159.707	690.837	1.26E+04	2.15E+03	8.10E+05	5.21E+03	6.00E+04	2.53E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	210	100	50	0.65	402.264	167.688	4.31E+03	1.33E+04	2.26E+03	1.35E+07	5.47E+03	1.54E+05	6.73E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	220	100	50	0.65	421.419	175.67	4.29E+03	1.39E+04	2.37E+03	1.41E+07	5.73E+03	1.59E+05	6.93E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	230	100	50	0.65	440.574	183.651	4.27E+03	1.45E+04	2.47E+03	1.47E+07	6.00E+03	1.64E+05	7.14E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	240	100	50	0.65	459.73	191.633	4.25E+03	1.52E+04	2.58E+03	1.52E+07	6.26E+03	1.69E+05	7.35E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	110	50	0.65	383.108	159.715	4.33E+03	1.26E+04	2.15E+03	1.27E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	120	50	0.65	383.108	159.723	4.33E+03	1.26E+04	2.15E+03	1.25E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	130	50	0.65	383.108	159.73	4.33E+03	1.26E+04	2.15E+03	1.22E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	140	50	0.65	383.108	159.738	4.33E+03	1.26E+04	2.15E+03	1.20E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	60	0.65	383.108	159.707	5.28E+03	1.26E+04	2.15E+03	1.39E+07	5.21E+03	1.58E+05	7.00E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	70	0.65	383.108	159.707	6.23E+03	1.26E+04	2.15E+03	1.48E+07	5.21E+03	1.68E+05	7.48E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	80	0.65	383.108	159.707	7.17E+03	1.26E+04	2.15E+03	1.56E+07	5.21E+03	1.77E+05	7.95E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	90	0.65	383.108	159.707	8.12E+03	1.26E+04	2.15E+03	1.65E+07	5.21E+03	1.87E+05	8.43E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.65	383.108	159.707	4.33E+03	1.26E+04	2.15E+03	1.30E+07	5.21E+03	1.48E+05	6.52E+04
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.75	423.4	176.443	9.09E+03	1.70E+04	2.91E+03	5.33E+07	7.04E+03	3.54E+05	1.55E+05
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.85	467.929	194.979	1.87E+04	2.26E+04	3.87E+03	2.18E+08	9.34E+03	8.28E+05	3.60E+05
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	0.95	517.142	215.479	3.81E+04	2.95E+04	5.07E+03	8.87E+08	1.22E+04	1.91E+06	8.28E+05
11	9	3	14	8	15	11	9	0.5	0.25	3	1	1	1	200	100	50	1.05	571.53	238.139	7.72E+04	3.82E+04	6.57E+03	3.60E+09	1.59E+04	4.37E+06	1.89E+06

IV DISCUSSION AND ANALYSIS

A. Observations with changing values of immigrant bacterial growth rate to stage-I plants

From the table-3.1, it is observed that $m_{100,}$ m_{010} and m_{001} are invariant functions of α_1; $m_{200,}$ m_{020} and m_{002} are increasing functions of α_1; $m_{110,}$ $m_{101,}$ and m_{011} are positive and increasing function of α_1. Hence it may conclude that the growth of bacteria through immigrations have no impact on the average sizes in each stage; the variances of three stages have increasing patterns with immigrated growth of bacteria; further there are positive and increasing correlations between the sizes of bacteria in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, influenced by the arrivals through immigrations of bacteria from outside the plants group.

B. Observations with changing values of internal growth of bacteria in stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} and (ii) m_{001} are increasing and decreasing functions respectively of β_1; further (i) $m_{200,}$ m_{020} and (ii) m_{002} are increasing and decreasing functions respectively of β_1; $m_{110,}$ $m_{101,}$ and m_{011} are positive and increasing functions of β_1. Hence it may conclude that there is a positive relation between average size of bacterial units in stage-I plants and the internal growth of bacteria in stage-I plants; positive relation between the internal growth rate of bacterial units in stage-I plants and the average size of bacteria in stage-II plants; negative relation between the internal growth rate of bacterial units in stage-I plants and the average size of bacteria in stage-III plants; the variances of bacterial units in first two stages are positively related and in third stage, negatively related with the internal growth rate of bacteria in stage-I plants; Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, influenced with the internal growth rate of plants in stage-I.

C. Observations with changing values of transition of bacteria from stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} and (ii) m_{100} are decreasing and increasing functions respectively of τ_1; further (i) $m_{200,}$ m_{020} and (ii) m_{002} are decreasing and increasing functions respectively of τ_1; $m_{110,}$ $m_{101,}$ and m_{011} are positive and decreasing function of τ_1. Hence it may conclude that there is a negative relation between average size of bacterial units in first two stages of plants and the transition of bacteria from stage-I plants to stage-II; positive relation between the average size of bacteria in stage-III plants and transition rate of bacterial units from stage-I to stage-II plants; the variances of bacterial units in first two stages of plants is negatively related and the variance of bacterial units in third stage plants is positively related with the transition rate of bacteria from stage-I plants to stage-II; Further, there is a positive and decreasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, plants influenced with the transition rate of bacteria from stage-I plants to stage-II.

D. Observations with changing values of immigrant bacterial growth rate to stage-II plants

From the table-3.1, it is observed that $m_{100,}$ m_{010} and m_{001} are invariant functions of α_2; further $m_{200,}$ m_{020} and m_{002} are also invariant functions of α_2; $m_{110,}$ $m_{101,}$ and m_{011} are positive and increasing function of α_2. Hence it may conclude that the growth of bacteria through immigrations have no impact on the average sizes in each stage; the variances of bacterial units in all three stages are not influenced by immigrated growth of bacteria; further there are positive and increasing correlations between the sizes of bacteria in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, influenced by the arrivals through immigrations of bacteria from outside the plants group.

E. Observations with changing values of internal growth of bacteria in stage-II plants

From the table-3.1, it is observed that (i) m_{100} and (ii) $m_{010,}$ m_{001} are invariant and decreasing functions respectively of β_2; further (i) m_{200} and (ii) $m_{020,}$ m_{002} are invariant and increasing functions respectively of β_2; (i) $m_{110,}$ m_{011} are positive and increasing function of β_2; and (ii) m_{101} is positive and decreasing function of β_2; Hence it may conclude that there is no impact of the internal growth of bacteria in stage-II plants on average size of bacterial units in stage-I plants and it has negative relation with the average size of bacteria in stage-II and stage-III plants; the variance of bacterial units in stage-I plants has no impact of the internal growth of bacteria in stage-II plants; the variance of bacterial units in stage-II and stage-III plants are positively related with the internal growth rate of bacteria in stage-II plants; Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-II and stage-III and positive and decreasing correlation between the sizes of the bacterial units in stage-I and stage-III plants influenced with the internal growth rate of plants in stage-II.

F. Observations with changing values of transition of bacteria from stage-II plants

From the table-3.1, it is observed that $m_{100,}$ m_{010} and m_{100} are invariant functions of τ_2; further $m_{200,}$ m_{020} and m_{002} are increasing functions of τ_2; $m_{110,}$ $m_{101,}$ and m_{011} are positive and increasing function of τ_2. Hence it may conclude that there is no influence on average size of bacterial units in all three stages of plants by the transition of bacteria from stage-II plants to stage-III; the variances of bacterial units in all three stages of plants are positively related with the transition rate of bacteria from stage-II plants to stage-III; Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, plants influenced with the transition rate of bacteria from stage-II plants to stage-III.

G. Observations with changing values of immigrant bacterial growth rate to stage-III

From the table-3.1, it is observed that $m_{100,}$ m_{010} and m_{001} are invariant functions of α_3; further (i) $m_{200,}$ m_{020} and (ii) m_{002} are invariant and increasing functions of α_3; (i) m_{110} is invariant (ii) m_{101} and m_{011} are positive and increasing function of α_3. Hence it may conclude that the growth of bacteria through immigrations in stage-III have no impact on the average sizes in each stage; the variance of bacterial units of stage-I and stage-II are not influenced by immigrated growth of bacteria; the variance of bacterial units of stage-III has increasing pattern with immigrated growth of bacteria; further there is no correlations between the sizes of bacteria in stage-I and stage-II; there is positive and increasing correlations between the sizes of bacteria in (i) stage-I and stage-III (ii) stage-II and stage-III, influenced by the arrivals through immigrations of bacteria from outside the plants group.

H. Observations with changing values of internal growth of bacteria in stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} and (ii) m_{001} are invariant and increasing functions respectively of β_3; further (i) $m_{200,}$ m_{020} and (ii) m_{002} are invariant and increasing functions respectively of β_3; (i) m_{110} is invariant (ii) $m_{101,}$ and m_{011} are positive and increasing functions of β_3. Hence it may conclude that there is no impact of the internal growth of bacteria in stage-III plants on the average size of bacterial units in first two stages; positive relation between average size of bacterial units in stage-III plants and the internal growth of bacteria in stage-III plants; the variances of bacterial units in first two stages are invariant and in third stage, positively related with the internal growth rate of bacteria in stage-III plants; Further, there is no correlation between the sizes of the bacterial units in stage-I and stage-II; a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-III (ii) stage-II and stage-III, influenced with the internal growth rate of plants in stage-III.

I. Observations with changing values of emigrant bacterial loss rate to stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} and (ii) m_{001} are decreasing and increasing functions respectively of ε_1; (i) $m_{200,}$ m_{020} and (ii) m_{002} are decreasing and increasing functions respectively of ε_1; $m_{110,}$ m_{101} and m_{011} is positive and decreasing function of ε_1; Hence it may conclude that the loss of bacteria through emigrations from stage-I plants have negative relation with the average sizes of bacteria in first two stages; positive relation with the average size of bacteria in third stage; the variance of bacterial units in first two stages of plants is negatively related with the loss of bacteria in stage-I plants; the variance of bacterial units in stage-III plants is positively related with the loss of bacteria in stage-I plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, influenced by the loss of bacteria from stage-I plants.

J. Observations with changing values of bacterial loss (death) rate in stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} and (ii) $m_{001}$are decreasing and increasing functions respectively of δ_1; (i) $m_{200,}$ m_{020} and (ii) m_{002} are decreasing and increasing functions respectively of δ_1; $m_{110,}$ m_{101} and m_{011} is positive and decreasing function of δ_1; Hence it may conclude that the loss of bacteria due to death in stage-I plants have (i) negative relation with the average sizes of bacteria in first two stages; (ii) positive relation with the average sizes of bacteria in third stage; the variance of bacterial units in first two stages of plants is negatively related with the loss (death) of bacteria in stage-I plants; the variance of bacterial units in stage-III plants is positively related with the loss (death) of bacteria in stage-I plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III, influenced by the loss (death) of bacteria from stage-I plants.

K. Observations with changing values of emigrant bacterial loss rate to stage-II plants

From the table-3.1, it is observed that (i) m_{100} is invariant (ii) m_{010} is decreasing and (iii) m_{001} is increasing function of ε_2; (i) m_{200} is invariant (ii) m_{020} and m_{002} are decreasing functions of ε_2; (i) m_{110} is positive and decreasing (ii) m_{101} is invariant (iii) m_{011} is positive and decreasing function of ε_2; Hence it may conclude that the loss of bacteria through emigrations from stage-II plants has no impact on the average size of bacteria in stage-I plants; the loss of bacteria through emigrations from stage-II plants has (i) negative relation with the average size of bacteria in stage-II plants (ii) positive relation with the average size of bacteria in stage-III plants; the variance of bacterial units in stage-I plants is not influenced by the loss of bacteria in stage-II plants; the variance of bacterial units in stage-II and stage-III plants are negatively related with the loss of bacteria in stage-II plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage -I and stage-II (ii) stage -II and stage-III; no correlation between the sizes of bacteria in stage-I and stage-III, influenced by the loss of bacteria from stage-II plants.

L. Observations with changing values of bacterial loss (death) rate in stage-II plants

From the table-3.1, it is observed that (i) m_{100} is invariant (ii) m_{010} is decreasing and (iii) m_{001} is increasing function of δ_2; (i) m_{200} is invariant (ii) m_{020} and m_{002} are decreasing functions of δ_2; (i) m_{110} is positive and decreasing (ii) m_{101} is invariant (iii) m_{011} is positive and decreasing function of δ_2; Hence it may conclude that the loss of bacteria due to death in stage-II plants has no impact on the average size of bacteria in stage-I plants; the loss (death) of bacteria in stage-II plants has (i) negative relation with the average size of bacteria in stage-II plants (ii) positive relation with the average size of bacteria in stage-III plants; the variance of bacterial units in stage-I plants is not influenced by the loss (death) of bacteria in stage-II plants; the variance of bacterial units in stage-II and stage-III plants are negatively related with the loss (death) of bacteria in stage-II plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage -I and stage-II (ii) stage -II and stage-III; no correlation between the sizes of bacteria in stage-I and stage-III, influenced by the loss (death) of bacteria from stage-II plants.

M. Observations with changing values of emigrant bacterial loss rate to stage-III plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} are invariant (ii) m_{001} is decreasing function of ε_3; (i) $m_{200,}$ m_{020} are invariant (ii) m_{002} is decreasing function of ε_3; (i) m_{110} is invariant (ii) $m_{101,}$ m_{011} are positive and decreasing function of ε_3; Hence it may conclude that the loss of bacteria through emigrations from stage-III plants has no impact on the average size of bacteria in first two stages of plants; the loss of bacteria through emigrations from stage-III plants has negative relation with the average size of bacteria in stage-III plants; the variance of bacterial units in first two stages of plants are not influenced by the loss of bacteria in stage-III plants; the variance of bacterial units in stage-III plants are negatively related with the loss of bacteria in stage-III plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage -I and stage-III (ii) stage -II and stage-III; no correlation between the sizes of bacteria in stage-I and stage-II, influenced by the loss of bacteria from stage-III plants.

N. Observations with changing values of bacterial loss (death) rate in stage-III plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} are invariant (ii) m_{001} is decreasing function of δ_3; (i) $m_{200,}$ m_{020} are invariant (ii) m_{002} is decreasing functions of δ_3; (i) m_{110} is invariant (ii) $m_{101,}$ m_{011} are positive and decreasing function of δ_3; Hence it may conclude that the loss of bacteria due to death in stage-III plants has no impact on the average size of bacteria in first two stages of plants; the loss (death) of bacteria in stage-III plants has negative relation with the average size of bacteria in stage-III plants; the variance of bacterial units in first two stages of plants are not influenced by the loss (death) of bacteria in stage-III plants; the variance of bacterial units in stage-III plants is negatively related with the loss (death) of bacteria in stage-III plants; further there is a positive and decreasing correlations between the sizes of bacteria in (i) stage-I and stage-III (ii) stage -II and stage-III; no correlation between the sizes of bacteria in stage-I and stage-II, influenced by the loss (death) of bacteria from stage-III plants.

O. Observations with changing values of initial number of bacterial units in stage-I plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} are increasing (ii) m_{001} is decreasing function of N_0; further, $m_{200,}$ m_{020} and m_{002} are increasing functions of N_0; $m_{110,}$ m_{101} and m_{011} are positive and increasing function of N_0. Hence it may conclude that there is a positive relation between the initial number of units of bacteria in stage-I and average size of bacterial units in first two stages; negative relation between the initial number of units of bacteria in stage-I and average size of bacterial units in stage-III; the variances of bacterial units in plants of all three stages are positively related with the initial number of units of bacteria in stage-I plants; Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III plants, influenced with the initial number of units of bacteria in stage-I.

P. Observations with changing values of initial number of bacterial units in stage-II plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{001} are invariant (ii) m_{010} is increasing function of M_0; further, (i) $m_{200,}$ m_{020} are invariant (ii) m_{002} are decreasing functions of M_0; $m_{110,}$ m_{101} and m_{011} are invariant functions of M_0. Hence it may conclude that there is no impact of initial number of bacteria in stage-II on average size of bacterial units in stage-I and Stage-III; positive relation between the initial number of units of bacteria in stage-II and average size of bacterial units in stage-II; the variances of bacterial units in first two stages of plants are not influenced by the initial number of units of bacteria in stage-II plants; the variances of bacterial units in stage-III plants are negatively related with the initial number of units of bacteria in stage-II plants; Further, there is a no correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III plants, influenced with the initial number of units of bacteria in stage-II.

Q. Observations with changing values of initial number of bacterial units in stage-III plants

From the table-3.1, it is observed that (i) $m_{100,}$ m_{010} are invariant (ii) m_{001} is increasing function of K_0; (i) $m_{200,}$ m_{020} are invariant (ii) m_{002} is increasing function of K_0; $m_{110,}$ m_{101} and m_{011} are positive and increasing functions of K_0. Hence it may conclude that there is no impact of initial number of bacteria in stage-III on average size of bacterial units in first two stages; positive relation between the initial number of units of bacteria in stage-III and average size of bacterial units in stage-III; the variances of bacterial units in plants of first two stages are not influenced by the initial number of units of bacteria in stage-III plants; the variance of bacterial units in stage-III is positively related with the initial number of units of bacteria in stage-III plants; Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III plants, influenced with the initial number of units of bacteria in stage-III.

R. Observations with changing values of time period

From the table-3.1, it is observed that $m_{100,}$ m_{010} and m_{001} are increasing functions of time t; $m_{200,}$ m_{020} and m_{002} are increasing functions of time t; $m_{110,}$ m_{101} and m_{011} is positive and increasing functions of time t. Hence it may conclude that there is a positive relation between average size of bacterial units in all three stages and the time; the variances of bacterial units in plants of all three stages are positively related with the time. Further, there is a positive and increasing correlation between the sizes of the bacterial units in (i) stage-I and stage-II (ii) stage-I and stage-III (iii) stage-II and stage-III plants, influenced by the time.

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