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Availability model for a consecutive-k-out-of-n:F system of non-identical components with fuzzy rates

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Abstract- In this paper availability model for a linear (circular) consecutive-k-out-of-n:F system (or C(k, n:F) system) of non-identical components has been considered. It is assumed that both the working time and repair time of each component are Rayleigh distributed. The estimate of the parameters from the Rayleigh distribution can be represented by triangular fuzzy numbers. A numerical example is solved to demonstrate the procedure clarifying the theoretical development**.**

Keywords - Linear (circular) C(k, n:F) system ; availability; Rayleigh distribution; fuzzy rates.

I. Introduction

Redundancy is used in design to improve system availability. A linear (circular) C(k, n :F) system is one kind of systems with redundancy. Suppose that n components are linearly (circularly) connected in such a way that the system fails if and only if at least k consecutive components fail. The C(k, n :F) system (both linear and circular) include the series and the parallel systems as special cases. For example, when k=n the linear and circular C(k, n :F) system becomes the series system, and when k=1, the linear and circular C(k, n :F) system becomes the parallel system. The problem of evaluating the reliability of the C(k, n:F) system has been the subject of many paper [1-5]. Though various studies of the C(k, n:F) system structure, have been reported in the literature, little attention has been paid to such systems that are repairable [6-10]. consecutive system. Sharifi and Moosakhani [12] works on a system with two element with constant and increasable fuzzy. Study a repairable consecutive-k-out-of-n:F system with fuzzy states introduced by Guan and Yueqin [13]. El-Damcese and Temraz [14] use a model for a k-out-of-n: F system that consists of n independent and identical components connected in parallel using non-homogeneous/ homogeneous continuous-time Markov chain.

This paper presents a procedure for computing intervals for the availability model for a linear (circular) C(k, n:F) system of non-identical components. An analysis of the life time and the repair time of the each component follow Rayleigh distribution. Fuzzy rates are used to model the uncertainty in the system availability. The estimated these fuzzy numbers using random samples.

II. AVAILABILITY OF COMPONENT

The transition diagram of the component failure rate $\lambda(t)$ and repair rate $\mu(t)$ is shown in Figure 1.

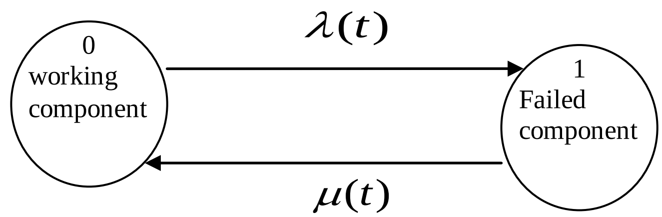

Figure 1. The transition diagram of the component

The Kolmogorov differential equation corresponding to continues time Markov chain of component availability associated with Figure 1 is

$$\frac{d}{dt}P_0(t) = -\lambda(t)P_0(t) + \mu(t)P_1(t) \tag{1}$$

Since $P_0(t) + P_1(t) = 1$, we have

$$\frac{d}{dt}P_0(t) = -(\lambda(t) + \mu(t))P_0(t) + \mu(t) \tag{2}$$

Consider that the life and repair times follow Rayleigh distribution, so the (failure/repair) rates are given by the following relation:

$$\lambda(t) = \frac{t}{\alpha^2} \text{ and } \mu(t) = \frac{t}{\beta^2} \qquad t \geq 0, \alpha, \beta \succ 0$$

Assuming $P_0(0) = 1$, then equation (2) can be solved to obtain the availability of component at time t:

$$A(t) = p_0(t) = \frac{1}{\alpha^2 + \beta^2}\{\alpha^2 + \beta^2$$

$$\times \exp[-\frac{1}{2}(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2})t^2]\} \qquad (3)$$

III. THE AVAILABILITY OF THE LINEAR (CIRCULAR) C(k, n:F) SYSTEM

The availability of the linear C(k, n:F) system composed of non-identical components associated with [15] is given by

$$\mathrm{A}_{line}(t;k,n)=\sum_{i=0}^{k-1}\sum_{j=r}^{m}[\prod_{l\in S^i_{b,j}}^{i}(1-A_l(t))][\prod_{l\in S^i_{g,j}}^{n-i}A_l(t)]$$
$$+\sum_{\mathrm{i=k}}^{\mathrm{M}}\sum_{j=r'}^{m'}[\prod_{l\in S^i_{b,j}}^{i}(1-A_l(t))][\prod_{l\in S^i_{g,j}}^{n-i}A_l(t)] \qquad (4)$$

with

$$A_l(t)=\frac{1}{\alpha_l^2+\beta_l^2}\{\alpha_l^2+\beta_l^2$$
$$\times \exp[-\frac{1}{2}(\frac{\alpha_l^2+\beta_l^2}{\alpha_l^2\beta_l^2})t^2]\} \qquad (5)$$

$$r=\sum_{z=0}^{i-1}\binom{n}{z},\ m=\sum_{z=1}^{i}\binom{n}{z},$$
$$r'=\sum_{z=0}^{i-1}N(z,k,n),\ m'=\sum_{z=1}^{i}N(z,k,n)$$

and

$$\mathrm{N(z,k,n)}=\begin{cases}0 & ,z>M\\ \binom{n}{z} & ,0\le z\le k-1\\ \sum_{w=0}^{\lfloor\frac{z}{k}\rfloor}(-1)^w\binom{n-z+1}{w}\binom{n-wk}{n-z} & ,k\le z\le n\end{cases} \qquad (6)$$

where

$A_l(t)$ is the availability of component *l at time t*,

$\mathrm{S}^{\mathrm{i}}_{\mathrm{b,j}}$ denote b^{th} bad component that belonging to state j with i bad components and $(\mathrm{n}-\mathrm{i})$ good components,

$\mathrm{S}^{\mathrm{i}}_{\mathrm{g,j}}$ denote g^{th} good component that belonging to state j with i bad components and $(\mathrm{n}-\mathrm{i})$ good components,

M is the maximum number of total components failures that a Linear C(k, n:F) system may experience without being failed,

$\binom{\mathrm{n}}{\mathrm{z}}$ number of combinations of z items out of possible n,

$\lfloor z/k \rfloor$ smallest integer less than or equal to z/k.

Where $\mathrm{A}_{\mathrm{line}}$(t; k, n) indicates the availability of the linear C(k, n:F) system. This equation reduces a circular system availability evaluation problem into a linear system availability evaluation problem.

For any $A_{line}(t;k,n-z-2), 0\le z\le k-1$ may be evaluated with equation (4) or other recursive equations to be introduced later.

The availability of the circular C(k, n:F) system is equal to the probability that there is a run of exactly i failures covering the selected point and the remaining n-z-2 components form a working linear C(k,n:F) system, where i may take values from 0 to k-1. A direct combinatorial approach is also available for the circular C(k, n:F) systems. Similar to equation (4) the following general equation can be used.

$$\mathrm{A}_{C}(t;k,n)=\sum_{i=0}^{k-1}\sum_{j=r}^{m}[\prod_{l\in S^i_{b,j}}^{i}(1-A_l(t))][\prod_{l\in S^i_{g,j}}^{n-i}A_l(t)]$$
$$+\sum_{\mathrm{i=k}}^{\mathrm{d}}\sum_{j=r''}^{m''}[\prod_{l\in S^i_{b,j}}^{i}(1-A_l(t))][\prod_{l\in S^i_{g,j}}^{n-i}A_l(t)] \qquad (7)$$

with

$$r''=\sum_{z=0}^{i-1}N_C(z,k,n),\ m''=\sum_{z=1}^{i}N_C(z,k,n)$$

Where d is the maximum number of failed components that may exist in the system without causing the system to fail and $N_C(z,k,n)$ is the number of ways of arranging n components including z failed ones in a circle such that at most k-1 failed ones are consecutive.

Using the same approach as for the linear system, we can derive the value of M as follows. If n is a multiple of k, then

$$d = \frac{n}{k}(k-1) = n - \frac{n}{k},$$

If n is not a multiple of k, then,

$$d = \left\lfloor \frac{n}{k} \right\rfloor (k-1) + (n-1-\left\lfloor \frac{n}{k} \right\rfloor k) = n-1-\left\lfloor \frac{n}{k} \right\rfloor$$

In summary, we have

$$d = \begin{cases} n - \frac{n}{k} & \text{if } n \text{ is a multiple of } k, \\ n - \left\lfloor \frac{n}{k} \right\rfloor - 1 & \text{if } n \text{ is not a multiple of } k. \end{cases} \quad (8)$$

the factor $N_C(z,k,n)$ can be expressed in terms of $N(z,k,n)$ which is for the linear systems, as given in the equation

$$N_C(z,k,n) = \frac{n}{n-z} N(z,\, k,\, n-1), \quad k \le z \le n-1 \quad (9)$$

IV. ESTIMATION OF THE (FAILURE/ REPAIR) RATES FOLLOW RAYLEIGH DISTRIBUTION

Consider that the life and repair times follow Rayleigh distribution, so the (failure/repair) rates with the MLE of the Rayleigh parameter are given by the following relation:

$$\hat{\lambda}_l(t) = \frac{t}{(\alpha_l^M)^2} \quad \text{and} \quad \hat{\mu}_l(t) = \frac{t}{(\beta_l^M)^2}$$

$$l = 1, 2, ..., n$$

where

$$\delta_l^M = \sqrt{\textstyle\sum_{i=1}^{q} X_i^{\,2}/2q} \qquad \delta = \alpha, \beta,\ l = 1, 2, .., n \quad (10)$$

If q, the size of random sample, is large ($q \ge 30$) then the $(1-\gamma)100\%$ confidence interval for each parameter α_l, β_l, $l = 1, 2, ..., n$ can be calculated from the following relations:

$$\left[\delta_l^L,\, \delta_l^U\right] = \left[\delta_l^M \mp Z_{\gamma/2}\sqrt{var\left(\delta_l^M\right)}\right], \quad \delta = \alpha, \beta,\ l = 1, 2, .., n \quad (11)$$

where

$$var\left(\delta_l^M\right) = \frac{(\delta_l^M)^2}{4q}$$

V. THE ESTIMATION OF FUZZY FAILURE AND REPAIR RATES

Usually, the failure rate and the repair rate of a component "l" have known probability distribution functions with parameters α_l, β_l driven from collected data or the opinions of the experts. Due to uncertainty in the values of these parameters, they can be modeled by triangular or trapezoidal fuzzy numbers. Here, we use the triangular membership functions $\tilde{\alpha}_l = (\tilde{\alpha}_l^L, \tilde{\alpha}_l^U), \tilde{\beta}_l = (\tilde{\beta}_l^L, \tilde{\beta}_l^U)$ which can written as follow:

$$\eta_{\tilde{\delta}_l}\left(x_l\right) = triangle\left(x_l;\, \delta_l^L,\, \delta_l^M,\, \delta_l^U\right) = \begin{cases} (x_l - \delta_l^L)/\left(\delta_l^M - \delta_l^L\right), & \delta_l^L \le x_l \le \delta_l^M \\ (\delta_l^U - x_l)/\left(\delta_l^U - \delta_l^M\right), & \delta_l^M \le x_l \le \delta_l^U \\ 0, \ \text{otherwise} \end{cases} \quad (12)$$

The fuzzy parameters $\tilde{\alpha}_l, \tilde{\beta}_l$ can be represented by crisp intervals, taking from the α-cuts of their membership functions; $0 < \alpha < 1$ as follow:

$$\tilde{\delta}_l(\alpha) = \left[\tilde{\delta}_l^L(\alpha),\, \tilde{\delta}_l^U(\alpha)\right]$$

$$= \left[\delta_l^L + \alpha\left(\delta_l^M - \delta_l^L\right),\, \delta_l^U - \alpha\left(\delta_l^U - \delta_l^M\right)\right] \quad (13)$$

VI. ILLUSTRATIVE EXAMPLE

In this example, consider the availability of linear (circular) C(2,4:F) system of non-identical components with life time and the repair time of this system follow Rayleigh probability distributions, The parameters can be represented by triangular fuzzy numbers.

The availability of linear C(2,4:F) system of non-identical components of equations (4) to (6) can be obtained from the following relation:

$$A_{line}(t;2,4) = \prod_{l=1}^{4} A_l(t) + \sum_{l=1}^{4}(1 - A_l(t))[\prod_{\substack{\nu=1 \\ \nu \ne l}}^{4} A_\nu(t)]$$

$$+A_1(t)A_3(t)[\prod_{\substack{\nu=2\\ \nu\neq 3}}^{4}(1-A_\nu(t))]+A_2(t)A_3(t)(1-A_1(t))$$
$$\times(1-A_4(t))+A_2(t)A_4(t)(1-A_1(t))(1-A_3(t))$$

The availability of circular C(2,4:F) system of non-identical components of equations (7) to (9) can be obtained from the following relation:

$$\mathrm{A}_C(t;2,4)=\prod_{l=1}^{4}A_l(t)+\sum_{l=1}^{4}(1-A_l(t))[\prod_{\substack{\nu=1\\ \nu\neq l}}^{4}A_\nu(t)]$$

$$+A_1(t)A_3(t)[\prod_{\substack{\nu=2\\ \nu\neq 3}}^{4}(1-A_\nu(t))]+A_2(t)A_4(t)(1-A_1(t))(1-A_3(t))$$

where

$$A_l(t)=\frac{1}{\alpha_l^2+\beta_l^2}\{\alpha_l^2+\beta_l^2$$
$$\times\exp[-\frac{1}{2}(\frac{\alpha_l^2+\beta_l^2}{\alpha_l^2\beta_l^2})t^2]\},\quad l=1,2,3,4$$

The statistical data of $\alpha_l, l=1,2,3,4$ taken from each component with γ= 0.05, 0.03, 0.07, 0.03 , q=70, 70, 50,50 and $\sum_{i=1}^{q}X_i^2 =$ 1220, 1100, 1000, 920 respectively , also the statistical data of $\beta_l, l=1,2,3,4$ taken from each component with γ= 0.075, 0.045, 0.065, 0.040 , q=36, 36, 40,40 and $\sum_{i=1}^{q}X_i^2 =$800,700, 850, 720 respectively.
The values of their lower, medium, and upper limits are calculated according to equations (10) and (11), we can obtain

$$\left(\alpha_1^L, \alpha_1^M, \alpha_1^U\right)=(2.606, 2.95, 3.296),$$
$$\left(\alpha_2^L, \alpha_2^M, \alpha_2^U\right)=(2.603, 3.028, 3.452)$$
$$\left(\alpha_3^L, \alpha_3^M, \alpha_3^U\right)=(2.757, 3.162, 3.567),$$
$$\left(\alpha_4^L, \alpha_4^M, \alpha_4^U\right)=(2.704, 3.197, 3.690)$$
And
$$\left(\beta_1^L, \beta_1^M, \beta_1^U\right)=(2.844, 3.333, 3.822),$$
$$\left(\beta_2^L, \beta_2^M, \beta_2^U\right)=(2.792, 3.416, 4.039)$$
$$\left(\beta_3^L, \beta_3^M, \beta_3^U\right)=(2.786, 3.260, 3.734),$$
$$\left(\beta_4^L, \beta_4^M, \beta_4^U\right)=(2.714, 3.303, 3.892)$$

According to the relation (13), we can obtain the ***α-cut** = 0, 0.1, 0.2 of the* $\left(\alpha_l^L, \alpha_l^U\right)$, and $\left(\beta_l^L, \beta_l^U\right)$, for $l=1,2,3,4$.

Table 1. The intervals for $\tilde{\alpha}_l$ and $\tilde{\beta}_l$ *for* $l=1,2,3,4$ corresponding to ***α-cut** = 0, 0.1, 0.2*

α-cut	$\left(\tilde{\alpha}_1^L, \tilde{\alpha}_1^U\right)$	$\left(\tilde{\alpha}_2^L, \tilde{\alpha}_2^U\right)$	$\left(\tilde{\alpha}_3^L, \tilde{\alpha}_3^U\right)$	$\left(\tilde{\alpha}_4^L, \tilde{\alpha}_4^U\right)$
0	[2.606, 3.298]	[2.603, 3.452]	[2.757, 3.567]	[2.704, 3.690]
0.1	[2.641, 3.26]	[2.646, 3.412]	[2.798, 3.409]	[2.753, 3.641]
0.2	[2.681, 3.23]	[2.698, 3.369]	[2.838, 3.369]	[2.803, 3.591]
α-cut	$\left(\tilde{\beta}_1^L, \tilde{\beta}_1^U\right)$	$\left(\tilde{\beta}_2^L, \tilde{\beta}_2^U\right)$	$\left(\tilde{\beta}_3^L, \tilde{\beta}_3^U\right)$	$\left(\tilde{\beta}_4^L, \tilde{\beta}_4^U\right)$
0	[2.844, 3.822]	[2.792, 4.039]	[2.786, 3.734]	[2.714, 3.892]
0.1	[2.894, 3.773]	[2.854, 3.967]	[2.833, 3.687]	[2.773, 3.833]
0.2	[2.942, 3.724]	[2.917, 3.914]	[2.881, 3.639]	[2.832, 3.774]

By using the intervals for $\tilde{\alpha}_l$ and $\tilde{\beta}_l$ corresponding to **α-cut**= 0, 0.1, 0.2 from Table 1, then represent graphically the two functions $\mathrm{A}_{line}(t;2,4)$ and $\mathrm{A}_C(t;2,4)$ at different values of **α-cut** as shown in Figures 2 and 3.

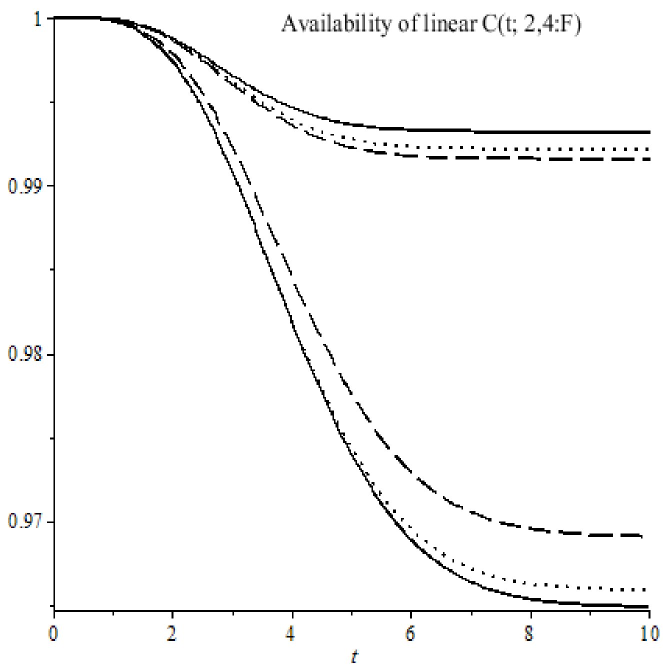

Figure 2. The $A_{line}(t;2,4)$ for **α-cut** = 0 (solid line), **α-cut**= 0.1 (dotted line),), **α-cut**= 0.2 (dashed line)

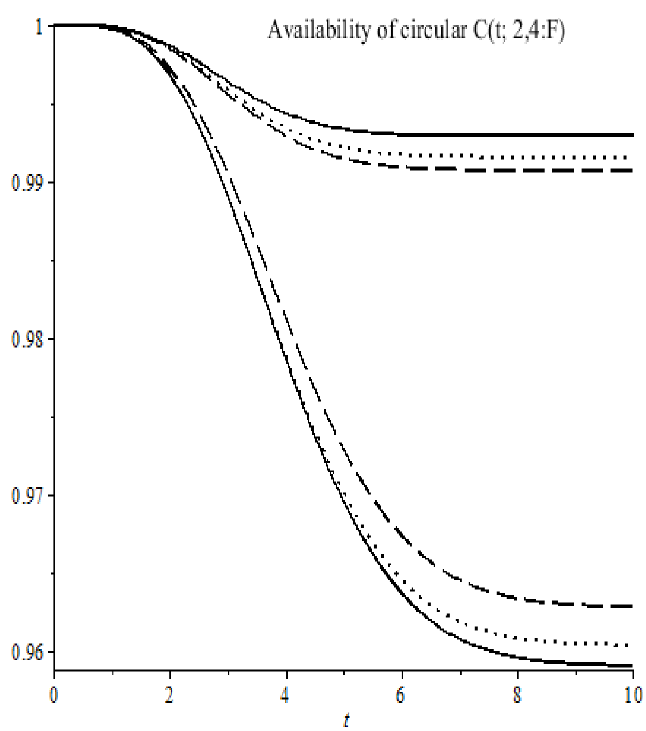

Figure 3. The $A_C(t;2,4)$ for **α-cut** = 0 (solid line), **α-cut**= 0.1 (dotted line), **α-cut**= 0.2 (dashed line)

VII. CONCLUSION

Consecutive-k-out-of-n system models have been proposed for system availability evaluation and the design of integrated circuits, microwave relay stations in telecommunications, oil pipeline systems, vacuum systems in accelerators, computer ring networks (k loop), and spacecraft relay stations. Such systems are characterized by logical or physical connections among components in lines or circles.

In this paper presents a model for the availability of linear (circular) C(k,n:F) system of independent and non- identical components. The estimated parameters of the Rayleigh distribution of the proposed model are fuzzy and all the fuzzy numbers have triangular membership functions. Fuzzy rates are used to model the uncertainty in the availability system. This paper includes a numerical example to illustrate the model.

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