International Journal of Scientific and Research Publications, Volume 5, Issue 3, March 2015 1
ISSN 2250-3153

Complement Properties on Strong Fuzzy Graphs

M.Vijaya

Department of Mathematics, Marudupandiyar College, Thanjavur, Tamil Nadu, India - 613403

Abstract- In this work we introduce the complement of strong fuzzy graph, tensor product of fuzzy graphs and strong fuzzy graph. Complement properties of tensor product of strong fuzzy graphs are discussed.

Index Terms- Complement of fuzzy graph, strong fuzzy graph, Tensor product.

I. Introduction

Rosenfeld [3] introduced fuzzy graph in 1975. The operations of Cartesian product, compositions of fuzzy graphs were defined by Moderson.J.N and peng.C.S [6]. In this note, we discuss a sub class of fuzzy graphs called strong fuzzy graph which were introduced by Moderson.J.N and peng.C.S [6]. In the present study we have been introduced the tensor product of fuzzy graphs. The operations on (crisp) graphs such as Cartesian product, composition, tensor, and normal products are extended to fuzzy graphs and some of their properties are incorporated to investigate. Properties found are related to complement of fuzzy graph and strong fuzzy graph

II. Basic definitions

Definition 2.1 Fuzzy graph with S as the underlying set is a pair G: (σ, μ) where σ: S → [0,1] is a fuzzy subset, μ:S x S→ [0,1] is a fuzzy relation on the fuzzy subset σ, such that μ(x,y) $\leq min$ {σ(x),σ(y)} for all for all x,y $\in$ S.

Definition 2.2 Let (σ, μ) be fuzzy sub graph of G = (V, X). Then (σ, μ) is called a strong [6] fuzzy graph of G if μ(u, v) = σ(u) ∧ σ(v) for all (u, v) ∈X.

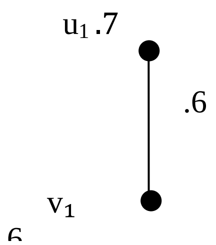


Fig. 1 Strong fuzzy graph

Definition 2.3 The complement of a fuzzy graph G^c : (σ^c,μ^c) where σ^c =σ and μ^c (uv)=0 when μ(uv)>0 μ^c (uv)=σ(u)Λσ(v) when μ(uv)=0 .

Note: $(G^c)^c$ = G if and only if G is a strong fuzzy graph.

Definition 2.4 The tensor Product of two fuzzy graphs (σ_i, μ_i) on $G_i = (V_i , X_i)$, i=1,2 is said to be a fuzzy graph $G_1 \otimes G_2$ = $(\sigma_1 \otimes \sigma_2 , \mu_1 \otimes \mu_2)$ on G = (V,X) where V= V_1 X V_2 and X ={$(u_1, u_2), (v_1, v_2)$\$(u_1,v_1) \in X_1,(u_2,v_2) \in X_2$}.
Fuzzy sets $\sigma_1 \otimes \sigma_2$ and $\mu_1 \otimes \mu_2$ are defined as
$(\sigma_1 \otimes \sigma_2)(u_1, u_2) = \{(\sigma_1(u_1) \wedge \sigma_2(u_2)\}$ for all $(u_1, u_2) \in V$ $(\mu_1 \otimes \mu_2)\{(u_1, u_2), (v_1, v_2)\} = \{\mu_1(u_1, v_1) \wedge \mu_2(u_2, v_2)\}$ $(u_1, u_2) \in X_1$ and $(v_1,v_2) \in X_2$

III. Properties

Tensor product of strong fuzzy graphs 3.1

The tensor product of two strong fuzzy graphs (σ_i, μ_i) on Gsi = (V_i , X_i), i=1,2 is defined as a strong fuzzy graph $\mathbf{G_{s_1} \otimes G_{s_2}}$= $(\sigma_1 \otimes \sigma_2 , \mu_1 \otimes \mu_2)$ on G = (V,X) where V= V_1 X V_2 and X ={$(u_1, u_2) (v_1, v_2)$\$(u_1,v_1) \in X_1 ,(u_2,v_2) \in x_2$}.
Fuzzy sets $\sigma = \sigma_1 \otimes \sigma_2$ and $\mu = \mu_1 \otimes \mu_2$ are defined as $\sigma(u_1, u_2) = (\sigma_1(u_1) \wedge \sigma_2(u_2))$ $\mu((u_1,v_1),(u_2,v_2)) = \{\sigma(u_1, v_1) \wedge \sigma(u_2,v_2)\}$

Theorem 3.2 Let G_{s1} :(σ_1, μ_1) and G_{s_2}: (σ_2, μ_2) be two strong fuzzy graphs. Then $G_{s_1} \otimes G_{s_2}$ is a strong fuzzy graph.
Let $G_{s_1} \otimes G_{s_2}$ = G:(σ,μ) where $\sigma = \sigma_1 \otimes \sigma_2$, $\mu = \mu_1 \otimes \mu_2$ and G* =(V,E) where
V =V_1 x V_2 ,E={$(u_1,u_2)(v_1,v_2)$:$u_1v_1 \in E_1$,$u_2v_2 \in E_2$}

Now, $\mu(u_1, u_2)(v_1,v_2) = \mu_1(u_1,v_1) \wedge \mu_2(u_2,v_2)$

$= (\sigma_1(u_1) \wedge \sigma_1(v_1)) \wedge (\sigma_2(u_2)\wedge\sigma_2(v_2))$ since G_1 and G_2 being strong.

$= (\sigma_1(u_1) \wedge (\sigma_2(u_2)\wedge\sigma_2(v_2))) \wedge (\sigma_1(v_1) \wedge (\sigma_2(u_2) \wedge \sigma_2(v_2)))$

$= (\sigma_1(u_1)\wedge(\sigma_2(u_2))\wedge(\sigma_1(u_1)\wedge\sigma_2(v_2))\}\wedge\{(\sigma_1(v_1)\wedge \sigma_2(u_2))\wedge(\sigma_1(v_1)\wedge\sigma_2(v_2))\}$

$= \{(\sigma_1\circ\sigma_2)(u_1, u_2)\wedge(\sigma_1\circ\sigma_2)(u_1,v_2)\}\wedge\{(\sigma_1\circ\sigma_2)(v_1,u_2)\wedge(\sigma_1\circ\sigma_2)(v_1,v_2)\}$

$=\{\sigma(u_1, u_2) \wedge \sigma(u_1,v_2)\}\wedge \{\sigma(v_1,u_2) \wedge \sigma(v_1,v_2)\}$

$= \sigma(u_1, u_2) \wedge \sigma(v_1, v_2)$.

Hence $G = G_{s_1} \otimes G_{s_2}$ is a strong fuzzy graph.

Theorem 3.3 If $G_{s1}:(\sigma_1, \mu_1)$ and $G_{s_2}: (\sigma_2, \mu_2)$ be two strong fuzzy graphs then $\overline{Gs_1 \otimes Gs_2} = \overline{Gs_1} \otimes \overline{Gs_2}$

Proof:

Let $G_{s1}:(\sigma_1, \mu_1)$ and $G_{s_2}: (\sigma_2, \mu_2)$ are strong fuzzy graphs. $\overline{G}$ $:(\sigma,\overline{\mu})= \overline{Gs_1 \otimes Gs_2}$ $\overline{\mu}=\overline{\mu_1 \otimes \mu_2}$, $\overline{G}: (V,\overline{E})$

$\overline{Gs_1}\ (\sigma_1, \overline{\mu_1})= \overline{G_1}\ (V_1, \overline{E_1})$
$\overline{Gs_2}(\sigma_2, \overline{\mu_2})= \overline{G_2}\ (V_2, \overline{E_2})$

$\overline{Gs_1} \otimes \overline{Gs_2} : (\sigma_1 \otimes \sigma_2, \overline{\mu_1} \otimes \overline{\mu_2})$

Now, the various types of edges say e, joining the vertices of V are the following and it suffices to prove that $\overline{\mu_1 \otimes \mu_2} = \overline{\mu_1} \otimes \overline{\mu_2}$ in each case.

Case (i)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1$, and $u_2v_2 \notin E_2$

Then $e \notin E$ Hence $\mu(e)=0$

Thus $\overline{\mu}(e) = \sigma(u_1, u_2) \wedge \sigma(v_1, v_2)$
$=[\sigma_1(u_1)\wedge(\sigma_2(u_2)]\wedge[(\sigma_1(u_1)\wedge\sigma_2(v_2)]$
$=[\sigma_1(u_1) \wedge \sigma_1(v_1)]\wedge [\sigma_2(u_2)\wedge\sigma_2(v_2)]$

Since $u_1v_1 \notin E_1 \Rightarrow u_1v_1 \in \overline{E_1}$ and $u_2v_2 \notin E_2 \Rightarrow$ $u_2v_2 \in \overline{E_2}$

we have $(\overline{\mu_1} \otimes \overline{\mu_2})(e) = \overline{\mu_1}(u_1v_1)\wedge \overline{\mu_2}(u_2v_2)$

$=[\sigma_1(u_1) \wedge \sigma_1(v_1)]\wedge [\sigma_2(u_2)\wedge\sigma_2(v_2)]$
$= \overline{\mu_1 \otimes \mu_2}(e)$

Case (ii)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \notin E_2$

Then $e \notin E$ also $e \notin \overline{E}$
Hence $\overline{\mu}(e)=0$
$u_1v_1 \notin E_1$, and $u_2v_2 \notin E_2$

$(\overline{\mu_1} \otimes \overline{\mu_2})(e)=0$

Case (iii)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \in E_2$
Then $e \notin E$ also $e \notin \overline{E}$

Hence $\overline{\mu}$ (e)=0

Also $u_1v_1 \in E_1, \Rightarrow u_1v_1 \notin \overline{E_1}$
$u_2v_2 \in E_2 \Rightarrow u_2v_2 \notin \overline{E_2}$
Hence $(\overline{\mu_1} \otimes \overline{\mu_2})(e) = 0$

Case (iv)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1,$ and $u_2v_2 \in E_2$

Then $e \notin E$ also $e \notin \overline{E}$
Hence $\overline{\mu}$ (e)=0

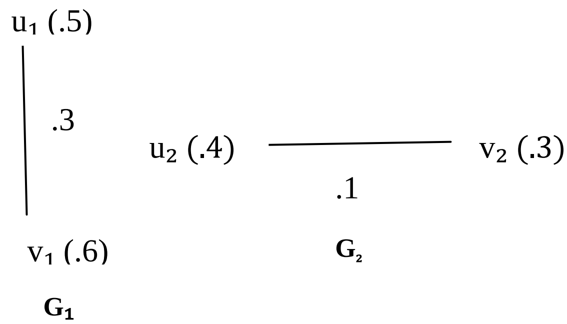
Also $u_1v_1 \notin E_1, \Rightarrow u_1v_1 \in \overline{E_1}$
$u_2v_2 \in E_2 \Rightarrow u_2v_2 \notin \overline{E_2}$
Hence $(\overline{\mu_1} \otimes \overline{\mu_2})(e) = 0$

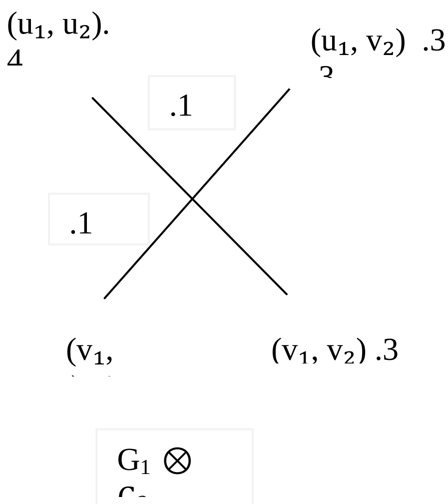
Thus from case (i) to (iv) it follows that $\overline{Gs_1 \otimes Gs_2} = \overline{Gs_1} \otimes \overline{Gs_2}$

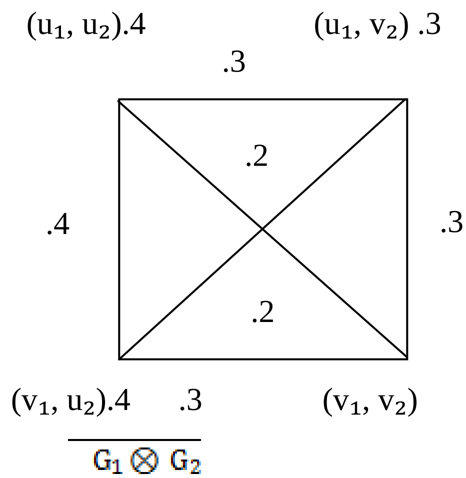
ISSN 2250-3153

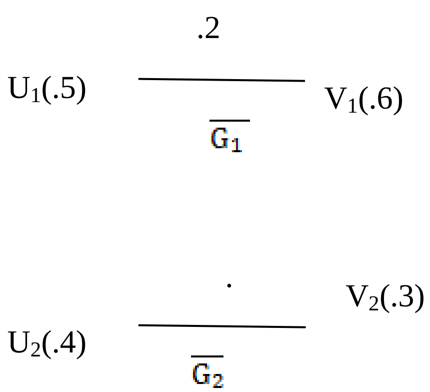
Remark 3.4

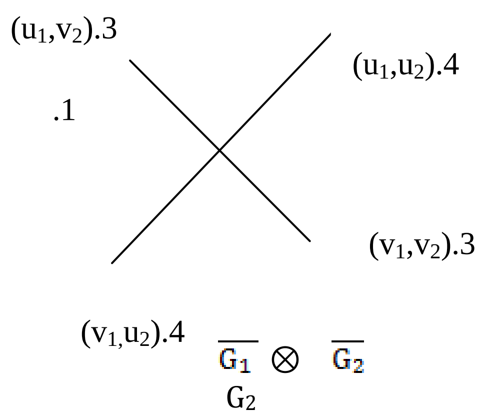
If general G_1 and G_2 are not strong fuzzy graphs then $\overline{G_1 \otimes G_2} \neq \overline{G_1} \otimes \overline{G_2}$



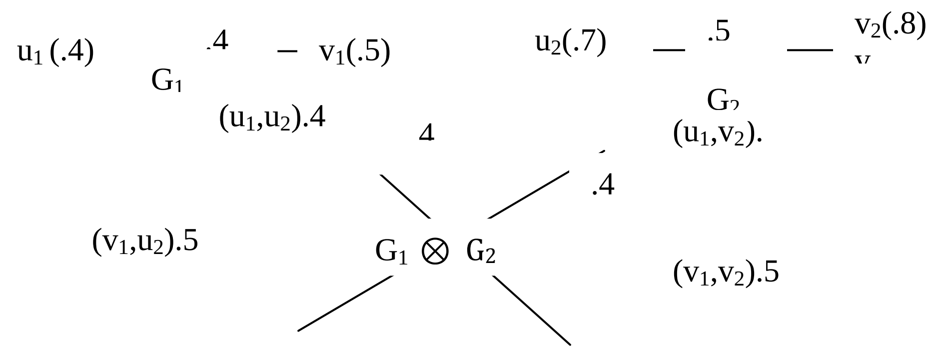




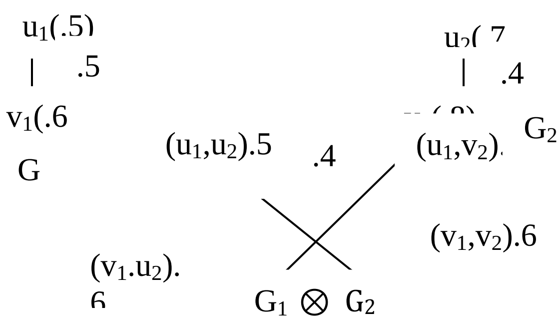




Corollary 3.5: If $\mathbf{G_1} \otimes G_2$ is strong then atleast G_1 or G_2 is strong.



In this example G_1 is strong G_2 is not strong and also leads that if one factor is strong then $\mathbf{G_1} \otimes G_2$ is strong. However, the following example shows that this may not always be true.



Here G_1 is strong G_2 is not strong then $\mathbf{G_1} \otimes G_2$ is not strong.

IV. Conclusion

In this paper we have proposed, complement of strong fuzzy graphs, tensor products of strong fuzzy graphs and the complement properties for tensor products of strong fuzzy graphs .In the fuzzy environment it is reasonable to discuss complement of strong fuzzy graphs and its properties.

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AUTHORS

First Author – Dr. M. Vijaya, Head, Department of Mathematics, Marudupandiyar College, Thanjavur-613403, Tamilnadu, India