On the cubic equation with four unknowns \( x^3 + y^3 = 14zw^2 \)

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Abstract - The sequences of integral solutions to the cubic equation with four variables are obtained. A few properties among the solutions are also presented.

Index Terms - Cubic equation having four unknowns with integral solutions.

I. INTRODUCTION

The Diophantine equation offers an unlimited field for research due to their variety [1-2]. In particular, one may refer [3-14] for cubic equations with three unknowns. In [15-18] cubic equations with four unknowns are studied for its non-trivial integral solutions. This communication concerns with the problem of obtaining non-zero integral solutions of cubic equation with four variables is given by \( x^3 + y^3 = 14zw^2 \). A few properties among the solutions and special numbers are presented.

Notations:

\[ t_{m,n} = n[1 + (n-1)(m-2)] \]
\[ p_n^m = \frac{n(n+1)}{6}[(n(m-2) + (5-m)] \]
\[ p_r^n = n(n+1) \]
\[ S_n = 6n(n-1) + 1 \]
\[ S_0 = n(2n^2 - 1) \]
\[ j_n = 2^n + (-1)^n \]
\[ j_n = \frac{1}{3}[2^n + (-1)^n] \]
\[ Gno_n = 2n - 1 \]
\[ CP_n^5 = n^3 \]

II. METHOD OF ANALYSIS

The cubic Diophantine equation with four unknowns to be solved for getting non-zero integral solution is

\[ x^3 + y^3 = 14zw^2 \]  \hspace{1cm} (1)

On substituting the linear transformations

\[ x = u + v, y = u - v, z = u \]  \hspace{1cm} (2)

in (1) leads to

\[ u^2 + 3v^2 = 7w^2 \]  \hspace{1cm} (3)

We obtain different choices of integral solutions to (1) through solving (3) which are illustrated as follows:

**Choice 1:**

Assume \( w = a^2 + 3b^2 \) \hspace{1cm} (4)

Write ‘7’ as \( 7 = (2 + i\sqrt{3})(2 - i\sqrt{3}) \) \hspace{1cm} (5)

Using (4) and (5) in (3) and employing factorization it is written as
\[ (u + i\sqrt{3}v)(u - i\sqrt{3}v) = (2 + i\sqrt{3})(2 - i\sqrt{3})(a + i\sqrt{3}b)^2(a - i\sqrt{3}b)^2 \]

Which is equivalent to the system of equations

\begin{align*}
(u + i\sqrt{3}v) &= (2 + i\sqrt{3})(a + i\sqrt{3}b)^2 \quad \text{(6a)} \\
(u - i\sqrt{3}v) &= (2 - i\sqrt{3})(a - i\sqrt{3}b)^2 \quad \text{(6b)}
\end{align*}

Equating the real and imaginary parts either in (6a) or (6b), we have

\begin{align*}
u &= 2a^2 - 6b^2 - 6ab \\
v &= a^2 - 3b^2 + 4ab
\end{align*}

In view of (2), the non-zero distinct integral solutions of (1) are

\begin{align*}
x &= 3a^2 - 9b^2 - 2ab \\
y &= a^2 - 3b^2 - 10ab \\
z &= 2a^2 - 6b^2 - 6ab
\end{align*}

along with (4)

Properties:

1) \( 3y(6a, a - 1) - x(6a, a - 1) + 32s - j_s - J_2 = 0 \)

2) \( 2w[a(a + 1), a + 2] - 2[a(a + 1), a + 2] - 12[Pr_3 + 3Pa^3 + 3a] + 1 \) is a perfect square.

3) \( 3y(a^2 + a, a + 3) - x(a^2 + a, a + 3) + 112t_{4, a} + 42(OH)_{a} \equiv 0 \text{ (mod 70)} \)

4) \( z(2b^2 + 1, b) - 2w(2b^2 + 1, b) + 12t_{4, a} + 18(OH)_{b} = 0 \)

Note:

In (5), 7 may also be considered as

\[ 7 = (-2 + i\sqrt{3})(-2 - i\sqrt{3}) \] (7)

For this case, the corresponding integer solutions are given by,

\begin{align*}
x &= -a^2 + 3b^2 - 10ab \\
y &= -3a^2 + 9b^2 - 2ab \\
z &= -2a^2 + 6b^2 - 6ab
\end{align*}

Choice 2:

Write ‘7’ as

\[ 7 = \frac{(5 + i\sqrt{3})(5 - i\sqrt{3})}{4} \] (8)

Using (4) and (8) in (3) and proceeding as in choice 1, the corresponding integer solutions are given by

\begin{align*}
x &= 3a^2 - 9b^2 + 2ab \\
y &= 2a^2 - 6b^2 - 8ab \\
z &= \frac{1}{2}[5a^2 - 15b^2 - 6ab]
\end{align*}

As our interest is of finding integral solutions, choose a and b suitably so that the solutions are in integers.

In particular, the choice \( a = 2A, b = 2B \) leads to the integer solutions to (1) are given by,
\[
x = 12A^2 - 36B^2 + 8AB \\
y = 8A^2 - 24B^2 - 32AB \\
z = 10A^2 - 30B^2 - 12AB \\
w = 4A^2 + 12B^2 \\
\]

**Properties:**

1) \[-x[a(a + 1), a + 2] + y[a(a + 1), a + 2] - w[a(a + 1), a + 2] + 8t_{4,a} - 8t_{4,a} + 240P_a^3 + 32P_a^5 = 0\]  
2) \[x(a, 1) + 3w(a, 1) - 8a \text{ is a nasty number.}\]  
3) \[x(a, 1) - y(a, 1) - w(a, 1) \equiv -24 \text{ (mod 40)}\]

Choose \[a = 2A + 1, b = 2B + 1\] leads to the integer solutions to (1) as

\[
x = 12A^2 - 36B^2 + 16A - 32B + 8AB - 4 \\
y = 8A^2 - 24B^2 - 8A - 40B - 32AB - 12 \\
z = 10A^2 - 30B^2 + 4A - 36B - 12AB - 8 \\
w = 14A^2 + 12B^2 + 4A + 12B + 4 \\
\]

**Properties:**

1) \[x(a, 1) - 4w(a, 1) + t_{10,a} + j_7 + J_7 \equiv -4 \text{ (mod 5)}\]  
2) \[z(a, 3) - t_{23,a} + 23a + 22 + J_6 \text{ is a cubical integer.}\]  
3) \[x(b + 1, b) - y(b + 1, b) - w(b + 1, b) + 24t_{4,b} - 40Pr_b \equiv 8 \text{ (mod 16)}\]

**Note:**

In (8), \(7\) can also be written as

\[
7 = \frac{(-5 + i\sqrt{3})(-5 - i\sqrt{3})}{4} \\
\]

For this case, the non-zero distinct are illustrated below,

For the choice \(a=2A, b=2B\)

\[
x = -8A^2 + 24B^2 - 32B \\
y = -12A^2 + 36B^2 + 8AB \\
z = -10A^2 + 30B^2 - 12AB \\
w = 4A^2 + 12B^2 \\
\]

For the choice \(a=2A+1, b=2B+1\)

\[
x = -8A^2 + 24B^2 - 24A + 8B - 32AB - 4 \\
y = -12A^2 + 36B^2 - 8A + 40B + 8AB + 8 \\
z = -10A^2 + 30B^2 - 16A + 24B - 12AB + 2 \\
w = 4A^2 + 12B^2 + 4A + 12B + 4 \\
\]

**Choice 3:**

Equation (3) can also be written as \(u^2 + 3v^2 = 7w^2 \times 1\)

Write’ 1’ as,
Using (4),(8) & (11) in (10) and employing the method of factorization as in choice 1, the corresponding integral solutions are given by
\[ x = 2a^2 - 6b^2 - 8ab, \]
\[ y = -a^2 + 3b^2 - 10ab, \]
\[ z = \frac{1}{2} \left[ a^2 - 3b^2 - 18ab \right] \]

As our aim is to find integral solutions, choose a and b suitably so that the solutions are in integers. In particular, the choice \( a = 2A, b = 2B \) leads to the integer solutions to (1) are given by,
\[ x = 8A^2 - 24B^2 - 32AB, \]
\[ y = -4A^2 + 12B^2 - 40AB, \]
\[ z = 2A^2 - 6B^2 - 36AB, \]
\[ w = 4A^2 + 12B^2 \]

Properties:
1) \( x(a-1,1) - 8t_{4,a} - Gao_a - j_a + 50a = 0 \)
2) \( z(a^2,a) + 36CP_a^6 - 2t_{4,a} \equiv 0 \pmod{6} \)
3) \( y(a+1,a) - 8t_{4,a} + 4Pr_a \equiv -4 \pmod{8} \)

The another choice \( a = 2A + 1, b = 2B + 1 \) leads to the integer solutions to (1) as given below
\[ x = 8A^2 - 24B^2 - 8A - 40B - 32AB - 12, \]
\[ y = -4A^2 + 12B^2 - 24A - 8B - 40AB - 8, \]
\[ z = 2A^2 - 6B^2 - 16A - 24B - 36AB - 10, \]
\[ w = 4A^2 + 12B^2 + 4A + 12B + 4 \]

Properties:
1) \( x(a,a^2-1) + 2y(a,a^2-1) + 56[Pr_a + (So)_a] \equiv 28 \pmod{56} \)
2) \( y(b+1,b^2) + w(b+1,b^2) - 24t_{4,b}^1 + 80P_b^1 - t_{10,b} \equiv -7 \pmod{17} \)
3) \( x[a(a+1),a+2] + 2y[a(a+1),a+2] + 56(Pr_a + 12P_a^1) \equiv -28 \pmod{56} \)

Note:
In (11), ‘1’ can also be written as
\[ 1 = \frac{(-1 + i\sqrt{3})(-1 - i\sqrt{3})}{4} \]  

For this case of ‘1’ the corresponding, non-zero distinct integer solutions are illustrated below
For the choice \( a = 2A, b = 2B \)
\[ x = -4A^2 + 12B^2 + 40AB \]
\[ y = 8A^2 - 24B^2 + 32AB \]
\[ z = 2A^2 - 6B^2 + 36AB \]
\[ w = 4A^2 + 12B^2 \]

For the choice a=2A+1, b=2B+1

\[ x = -4A^2 + 12B^2 + 16A + 32B + 40AB + 12 \]
\[ y = 8A^2 - 24B^2 + 24A - 8B + 32AB + 4 \]
\[ z = 2A^2 - 6B^2 + 20A + 12B + 36AB + 8 \]
\[ w = 4A^2 + 12B^2 + 4A + 12B + 4 \]

**Choice 4:**

'1' can also be written as,

\[ 1 = \frac{(1 + i4\sqrt{3})(1 - i4\sqrt{3})}{49} \]  
(13)

Using (4),(5) & (13) in (10) and employing the method of factorization as in choice 1, the corresponding integer solutions are given by

\[ x = \frac{1}{7}[-a^2 + 3b^2 - 74ab] \]
\[ y = \frac{1}{7}[-19a^2 + 57b^2 - 34ab] \]
\[ z = \frac{1}{7}[-10a^2 + 30b^2 - 54ab] \]

As our interest is of finding integer solutions, choose a and b suitably so that the solutions are in integers.

By taking a=7A, b=7B leads to the integer solutions to (1) to be,

\[ x = -7A^2 + 21B^2 - 518AB \]
\[ y = -133A^2 + 399B^2 - 238AB \]
\[ z = -70A^2 + 210B^2 - 378AB \]
\[ w = 49A^2 + 147B^2 \]

**Properties:**

1) \[ 19x(a, a^2 - 1) - y(a, a^2 - 1) + 4802(So)_a \equiv 0 \pmod{4802} \]
2) \[ 19x(7a - 5, a) - y(7a - 5, a) + 19208t_{9,a} = 0 \]
3) \[ w(1, b) + 7x(1, b) - t_{90,b} \equiv 0 \pmod{3333} \]

**Note:**

In (13), ‘1’ can also be written as

\[ 1 = \frac{(-1 + i4\sqrt{3})(-1 - i4\sqrt{3})}{49} \]  
(14)

For the choice, the integer solutions are
\[ x = -133A^2 + 399B^2 + 238AB \]
\[ y = -7A^2 + 21B^2 + 518AB \]
\[ z = -70A^2 + 210B^2 + 378AB \]
\[ w = 49A^2 + 147B^2 \]

**Choice 5:**

Equation (3) can be re-written as
\[ u^2 - 4w^2 = 3(w^2 - v^2) \]
which is written in the form of ratio as,
\[ \frac{u + 2w}{w - v} = \frac{3(w + v)}{u - 2w} = \frac{a}{b} \] (15)

which is equivalent to the system of equations,
\[ bu + av + (2b - a)w = 0 \]
\[ au - 3bv - (2a + 3b)w = 0 \]

Applying the method of cross multiplication we have,
\[ u = -2a^2 + 6b^2 - 6ab \]
\[ v = -a^2 + 3b^2 + 4ab \]
\[ w = a^2 + 3b^2 \]

Substituting the values of ‘u’ and ‘v’ we get the non-zero distinct integral solutions to be
\[ x = -3a^2 + 9b^2 - 2ab \]
\[ y = -a^2 + 3b^2 - 10ab \]
\[ z = -2a^2 + 6b^2 - 6ab \]

**Properties:**

1) \[ 2w(2a, a^2) + z(2a, a^2) - 12(t_{4, a^2} + CP_a^6) = 0 \]
2) \[ x(2b - 1, b) - 3y(2b - 1, b) - 28bGbo_b = 0 \]
3) \[ z(a^2, a) - 2y(a^2, a) - 14CP_a^6 = 0 \]

**Choice 6:**

(15) can also be written as,
\[ \frac{u + 2w}{3(w - v)} = \frac{(w + v)}{u - 2w} = \frac{a}{b} \]
which is equivalent to the system of equations,
\[ bu + 3av + (2b - 3a)w = 0 \]
\[ au - bv - (2a + b)w = 0 \]

Applying the method of cross multiplication we have,
\[
\begin{align*}
\text{In view of (2), we get the non-zero distinct integer solutions to be} \\
x &= -9a^2 + 3b^2 - 2ab \\
y &= -3a^2 + b^2 - 10ab \\
z &= -6a^2 + 2b^2 - 6ab \\
\end{align*}
\]

**Properties:**

1) \(y(a, a^2) + z(a, a^2) - 3t_{a,a} + 16CP_a^6 + 9a = 0\)

2) \(x(a + 1, a) - 3w(a + 1, a) - 6t_{a,a} + 2 Pr_a = 0\)

3) \(-10y(a, a) \text{ and } -8x(a, a) \) are perfect squares.

Now instead of (2), writing the linear transformations as

\[
\begin{align*}
x &= u + v, \\
y &= u - v, \\
z &= 4u
\end{align*}
\]

in (1), it leads to

\[
u^2 + 3v^2 = 28w^2
\]

Solving (17) in different ways one obtains other different choices of integer solutions to (1) which are illustrated as below:

**Choice 7:**

Write ‘28’ as

\[
28 = (5 + i\sqrt{3})(5 - i\sqrt{3})
\]

Using (4) and (18) in (17) and proceeding as in choice 1, the corresponding integer solutions are given by

\[
\begin{align*}
x &= 6a^2 - 18b^2 + 4ab \\
y &= 4a^2 - 12b^2 - 16ab \\
z &= 24a^2 - 72b^2 - 16ab \\
w &= a^2 + 3b^2
\end{align*}
\]

**Properties:**

1) \(x(a, 2a - 1) + 6w(a, 2a - 1) - 12t_{a,a} - 4t_{a,a} = 0\)

2) \(6y(6b, b - 1) - z(6b, b - 1) + 80S_b \equiv 0 \text{ (mod 80)}\)

3) \(x[a + 1, a(a + 2)] + 6w[a + 1, a(a + 2)] - 12t_{a,a} - 24P_a^3 \equiv 12 \text{ (mod 24)}\)

4) \(6y[1, a(2a^2 + 1) - z[1, a(2a^2 + 1)] + 240(OH)_a = 0\)

**Note:**

In (18), ‘28’ can also be considered as

\[28 = (-5 + i\sqrt{3})(-5 - i\sqrt{3})\]

For this choice the corresponding non-zero distinct integer solutions are obtained as
\[
x = -4a^2 + 12b^2 - 16ab \\
y = -6a^2 + 18b^2 + 4ab \\
z = -20a^2 + 60b^2 - 24ab \\
w = a^2 + 3b^2
\]

**Choice 8:**

Consider ‘28’ as
\[
28 = (4 + i2\sqrt{3})(4 - i2\sqrt{3}) \tag{19}
\]

Using (4) and (19) in (17) and proceeding as in choice 1, the integer solutions are as follows
\[
x = 6a^2 - 18b^2 - 4ab \\
y = 2a^2 - 6b^2 - 20ab \\
z = 16a^2 - 48b^2 - 48ab \\
w = a^2 + 3b^2
\]

**Properties:**

1) \[5x(a,1) - y(a,1) - 28t_{4,a} + 84 = 0\]
2) \[y(2b^2 - 1, b) - 2w(2b^2 - 1, b) + 20(SO)_b + 12Pr_b \equiv 0 \pmod{12}\]
3) \[x[a(a + 1), a + 2] - 3y[a(a + 1), a + 2] - 336P_a^3 = 0\]
4) \[z(6a, a - 1) + 16w(6a, a - 1) + 48(Sa - 1) - 1152t_{4,a} = 0\]

**Note:**

In (19), ‘28’ can be written as
\[
28 = (-4 + i2\sqrt{3})(-4 - i2\sqrt{3}) \tag{20}
\]

By choosing this, the non-zero distinct integer solutions are given by
\[
x = -2a^2 + 6b^2 - 20ab \\
y = -6a^2 + 18b^2 - 4ab \\
z = -16a^2 + 48b^2 - 48ab \\
w = a^2 + 3b^2
\]

**Choice 9:**

(17) can also be written as
\[
u^2 + 3v^2 = 28w^2 \ast 1 \tag{21}
\]

Using (14), (19) and (11) in (21) and proceeding as in pattern 1, the corresponding integral solutions are given by
\[
x = 2a^2 - 6b^2 - 20ab \\
y = -4a^2 + 12b^2 - 16ab \\
z = -4a^2 + 12b^2 - 72ab \\
w = a^2 + 3b^2
\]

**Properties:**

1) \[x[1, a(2a^2 - 1)] + 2w[1, a(2a^2 - 1)] + 20(SO)_a \text{ is a perfect square.}\]
2) \[ y[6a(a - 1), 1] + 4w[6a(a - 1), 1] + 16(S_a - 1) \] is a nasty number.

3) \[ y(b + 1, b + 2) + 4w(b + 1, b + 2) - S_b - 2t_{4,b} \equiv 9 \, \text{(mod 54)} \]

4) \[ y[a^2 + a, (a + 2)(a + 3)] - z[a^2 + a, (a + 2)(a + 3)] + 1344P_{a} = 0 \]

**Note:**
Using (4), (12) and (20) in (21), we have the following set of solutions satisfying (1).

\[
\begin{align*}
  x &= -4a^2 + 12b^2 + 16ab \\
  y &= 2a^2 - 6b^2 + 20ab \\
  z &= -4a^2 + 12b^2 + 72ab \\
  w &= a^2 + 3b^2
\end{align*}
\]

**Choice 10:**
Using (4), (13) and (19) in (21) and employing the method of factorization we have the distinct integer solutions are as follows;

\[
\begin{align*}
  x &= \frac{1}{7}[-2a^2 + 6b^2 - 148ab] \\
  y &= \frac{1}{7}[-38a^2 + 114b^2 - 68ab] \\
  z &= \frac{1}{7}[-80a^2 + 240b^2 - 432ab]
\end{align*}
\]

As our aim is to find integral solutions choose \( a \) and \( b \) suitably so that the solutions are integers.
Taking \( a = 7A, b = 7B \)

\[
\begin{align*}
  x &= -14A^2 + 42B^2 - 1036AB \\
  y &= -266A^2 + 798B^2 - 476AB \\
  z &= -560A^2 + 1680B^2 - 3024AB \\
  w &= 49A^2 + 147B^2
\end{align*}
\]

**Properties:**

\[
\begin{align*}
  1) & \quad z(a^2 + a, 2a^2 - 1) - 40x(a^2 + a, 2a^2 - 1) + 38416P_{a} - 76832t_{4,a}P_{a} = 0 \\
  2) & \quad 7x(b + 1, b) + 2w(b + 1, b) - 588t_{4,b} + 7252P_{b} = 0 \\
  3) & \quad 19x(a, 2a^2 + 1) - y(a, 2a^2 + 1) + 57624(OH)_{a} = 0 \\
  4) & \quad 19x(a(a + 1), a + 2) - y(a(a + 1), a + 2) + 115248P_{a}^3 = 0
\end{align*}
\]

**Note:**
Using (4), (14) and (20) in (21), we have the distinct integral solutions are obtained below

\[
\begin{align*}
  x &= -266A^2 + 798B^2 + 476AB \\
  y &= -14A^2 + 42B^2 + 1036AB \\
  z &= -560A^2 + 1680B^2 + 3024AB \\
  w &= 49A^2 + 147B^2
\end{align*}
\]

**Choice 11:**
(17) can be rewritten as

\[ u^2 - 25w^2 = 3(w^2 - v^2) \]

which is written in the form of ratio as
\[
\frac{u + 5w}{w + v} = \frac{3(w - v)}{u - 5w} = \frac{a}{b}
\]  \hspace{1cm} (23)

which is equivalent to the system of equations,

\[
\begin{align*}
bu - av + (5b - a)w &= 0 \\
u a + 3bv - (3b + 5a)w &= 0
\end{align*}
\]

Applying the method of cross multiplication we have,

\[
\begin{align*}
u &= 5a^2 - 15b^2 + 6ab \\
v &= -a^2 + 3b^2 + 10ab \\
w &= a^2 + 3b^2
\end{align*}
\]

Substituting the values of \(u, v\) in (16) we get the non-zero distinct integer solutions to (1) to be

\[
\begin{align*}
x &= 4a^2 - 12b^2 + 16ab \\
y &= 6a^2 - 18b^2 - 4ab \\
z &= 20a^2 - 60b^2 + 24ab \\
w &= 3a^2 + b^2
\end{align*}
\]

**Properties:**

1) Each of the following expressions represents the nasty number.

- a) \(3\{x(a, 2a^2 + 1) + 4w(a, 2a^2 + 1) - 48(OH)\}
- b) \(6y(b^2 + b, b) + z(b^2 + b, b) + 224P^5 - 2b^4\)
- c) \(3\{x(a, a^2 + a) + 4w(a, a^2 + a) - 32P^5\}\)

2) \(6w(2b^2 - 1, b) - y(2b^2 - 1, b) - 4(SO)_b\) is a perfect square.

**Choice 12:**

(23) can also be written as
\[
\frac{u + 5w}{w + v} = \frac{(w - v)}{u - 5w} = \frac{a}{b}
\]

which is equivalent to the system of equations,

\[
\begin{align*}
bu - 3av + (5b - 3a)w &= 0 \\
u a + bv - (5a + b)w &= 0
\end{align*}
\]

Applying the method of cross multiplication we have,

\[
\begin{align*}
u &= 15a^2 - 5b^2 + 6ab \\
v &= -3a^2 + b^2 + 10ab \\
w &= 3a^2 + b^2
\end{align*}
\]

In view of (16) we get the non-zero distinct integer solutions to (1) are

\[
\begin{align*}
x &= 12^2 - 4b^2 + 16ab \\
y &= 18a^2 - 6b^2 - 4ab \\
z &= 60a^2 - 20b^2 + 24ab \\
w &= 3a^2 + b^2
\end{align*}
\]
Properties:

1) Each of the following expressions represents a perfect square:
   a) \(2y(a,a)z(a,a)\)
   b) \(x(a+1,a) + t_{k,a} - 16Pr_{k,a} + 4\)

2) \(x(a,a)\) and \(4w(a,a)\) represents nasty numbers.

III. Conclusion

One may search for other Choices of solutions and their corresponding properties.

REFERENCES


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