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On the cubic equation with four unknowns $x^3 + y^3 = 14zw^2$

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Abstract- The sequences of integral solutions to the cubic equation with four variables are obtained. A few properties among the solutions are also presented.

Index Terms- Cubic equation having four unknowns with integral solutions.

I. Introduction

The Diophantine equation offer an unlimited field for research due to their variety [1-2]. In particular, one may refer [3-14] for cubic equation with three unknowns. In [15-18] cubic equations with four unknowns are studied for its non-trivial integral solutions. This communication concerns with the problem of obtaining non-zero integral solutions of cubic equation with four variables is given by $x^3 + y^3 = 14zw^2$. A few properties among the solutions and special numbers are presented.

Notations:

$$t_{m,n} = n[1 + \frac{(n-1)(m-2)}{2}]$$

$$P_n^m = \frac{n(n+1)}{6}[n(m-2) + (5-m)]$$

$$\mathrm{Pr}_n = n(n+1)$$

$$S_n = 6n(n-1) + 1$$

$$So_n = n(2n^2 - 1)$$

$$j_n = 2^n + (-1)^n$$

$$J_n = \frac{1}{3}[2^n + (-1)^n]$$

$$Gno_n = 2n - 1$$

$$CP_n^6 = n^3$$

II. Method of analysis

The cubic Diophantine equation with four unknowns to be solved for getting non-zero integral solution is

$$x^3 + y^3 = 14zw^2 \qquad (1)$$

On substituting the linear transformations

$$x = u + v, y = u - v, z = u \qquad (2)$$

in (1) leads to

$$u^2 + 3v^2 = 7w^2 \qquad (3)$$

We obtain different choices of integral solutions to (1) through solving (3) which are illustrated as follows:

Choice 1:

Assume , $w = a^2 + 3b^2$ (4)

Write '7' as $7 = (2 + i\sqrt{3})(2 - i\sqrt{3})$ (5)

Using (4)and(5) in (3) and employing factorization it is written as

$$(u+i\sqrt{3}v)(u-i\sqrt{3}v)=(2+i\sqrt{3})(2-i\sqrt{3})(a+i\sqrt{3}b)^2(a-i\sqrt{3}b)^2$$

Which is equivalent to the system of equations

$$(u+i\sqrt{3}v)=(2+i\sqrt{3})(a+i\sqrt{3}b)^2 \quad (6a)$$

$$(u-i\sqrt{3}v)=(2-i\sqrt{3})(a-i\sqrt{3}b)^2 \quad (6b)$$

Equating the real and imaginary parts either in (6a) or (6b), we have

$$u=2a^2-6b^2-6ab$$

$$v=a^2-3b^2+4ab$$

In view of (2), the non-zero distinct integral solutions of (1) are

$$x=3a^2-9b^2-2ab$$

$$y=a^2-3b^2-10ab$$

$$z=2a^2-6b^2-6ab$$

along with (4)

Properties :

1) $3y(6a,a-1)-x(6a,a-1)+32s_a-j_5-J_2=0$

2) $2w[a(a+1),a+2]-z[a(a+1),a+2]-12[\Pr_a+3Pa^3+3a]+1$ is a perfect square.

3) $3y(a^2+a,a+3)-x(a^2+a,a+3)+112t_{4,a}+42(OH)_a\equiv 0(\bmod 70)$

4) $z(2b^2+1,b)-2w(2b^2+1,b)+12t_{4,b}+18(OH)_b=0$

Note :

In (5), 7 may also be considered as

$$7=(-2+i\sqrt{3})(-2-i\sqrt{3}) \quad (7)$$

For this case, the corresponding integer solutions are given by ,

$$x=-a^2+3b^2-10ab$$

$$y=-3a^2+9b^2-2ab$$

$$z=-2a^2+6b^2-6ab$$

Choice 2:

Write' 7' as $7=\dfrac{(5+i\sqrt{3})(5-i\sqrt{3})}{4}$ (8)

Using (4) and (8) in (3) and proceeding as in choice 1, the corresponding integer solutions are given by

$$x=3a^2-9b^2+2ab$$

$$y=2a^2-6b^2-8ab$$

$$z=\frac{1}{2}[5a^2-15b^2-6ab]$$

As our interest is of finding integral solutions, choose a and b suitably so that the solutions are in integers. In particular, the choice a=2A, b=2B leads to the integer solutions to (1) are given by,

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$x = 12A^2 - 36B^2 + 8AB$

$y = 8A^2 - 24B^2 - 32AB$

$z = 10A^2 - 30B^2 - 12AB$

$w = 4A^2 + 12B^2$

Properties:

1) $-x[a(a+1), a+2] + y[a(a+1), a+2] - w[a(a+1), a+2] + 8t_{4,a^2} - 8t_{4,a} + 240P_a^3 + 32P_a^5 = 0$ **2)** $x(a,1) + 3w(a,1) - 8a$ is a nasty number.

3) $x(a,1) - y(a,1) - w(a,1) \equiv -24 \pmod{40}$

Choose $a = 2A+1, b = 2B+1$ leads to the integer solutions to (1) as

$x = 12A^2 - 36B^2 + 16A - 32B + 8AB - 4$

$y = 8A^2 - 24B^2 - 8A - 40B - 32AB - 12$

$z = 10A^2 - 30B^2 + 4A - 36B - 12AB - 8$

$w = 14A^2 + 12B^2 + 4A + 12B + 4$

Properties:

1) $x(a,1) - 4w(a,1) + t_{10,a} + j_7 + J_7 \equiv -4 \pmod{5}$

2) $z(a,3) - t_{22,a} + 23a + 22 + J_6$ is a cubical integer.

3) $x(b+1,b) - y(b+1,b) - w(b+1,b) + 24t_{4,b} - 40\,\mathrm{Pr}_b \equiv 8 \pmod{16}$

Note:

In (8),' 7' can also be written as

$$7 = \frac{(-5+i\sqrt{3})(-5-i\sqrt{3})}{4} \tag{9}$$

For this case, the non-zero distinct are illustrated below,

For the choice a=2A,b=2B

$x = -8A^2 + 24B^2 - 32B$

$y = -12A^2 + 36B^2 + 8AB$

$z = -10A^2 + 30B^2 - 12AB$

$w = 4A^2 + 12B^2$

For the choice a=2A+1,b=2B+1

$x = -8A^2 + 24B^2 - 24A + 8B - 32AB - 4$

$y = -12A^2 + 36B^2 - 8A + 40B + 8AB + 8$

$z = -10A^2 + 30B^2 - 16A + 24B - 12AB + 2$

$w = 4A^2 + 12B^2 + 4A + 12B + 4$

Choice 3:

Equation (3) can also be written as $u^2 + 3v^2 = 7w^2 * 1$ (10)

Write' 1' as,

$$1=\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \qquad (11)$$

Using (4),(8) & (11) in (10) and employing the method of factorization as in choice 1,the corresponding integral solutions are given by

$$x=2a^2-6b^2-8ab$$

$$y=-a^2+3b^2-10ab$$

$$z=\frac{1}{2}[a^2-3b^2-18ab]$$

As our aim is to find integral solutions, choose a and b suitably so that the solutions are in integers.
In particular, the choice a=2A, b=2B leads to the integer solutions to (1) are given by,

$$x=8A^2-24B^2-32AB$$

$$y=-4A^2+12B^2-40AB$$

$$z=2A^2-6B^2-36AB$$

$$w=4A^2+12B^2$$

Properties:

1) $x(a-1,1)-8t_{4,a}-Gao_a-j_4+50a=0$

2) $z(a^2,a)+36CP_a^6-2t_{4,a}\equiv 0(\bmod 6)$

3) $y(a+1,a)-8t_{4,a}+40\Pr_a\equiv -4(\bmod 8)$

The another choice $a=2A+1, b=2B+1$ leads to the integer solutions to (1) as given below

$$x=8A^2-24B^2-8A-40B-32AB-12$$

$$y=-4A^2+12B^2-24A-8B-40AB-8$$

$$z=2A^2-6B^2-16A-24B-36AB-10$$

$$w=4A^2+12B^2+4A+12B+4$$

Properties:

1) $x(a,a^2-1)+2y(a,a^2-1)+56[\Pr_a+(So)_a]\equiv 28(\bmod 56)$

2) $y(b+1,b^2)+w(b+1,b^2)-24t_{4,b^2}+80P_b^5-t_{10,b}\equiv -7(\bmod 17)$

3) $x[a(a+1),a+2]+2y[a(a+1),a+2]+56(\Pr_a+12P_a^3)\equiv -28(\bmod 56)$

Note:

In (11), ‘1’ can also be written as

$$1=\frac{(-1+i\sqrt{3})(-1-i\sqrt{3})}{4} \qquad (12)$$

For this case of ‘1’ the corresponding, non-zero distinct integer solutions are illustrated below
For the choice a=2A,b=2B

$$x = -4A^2 + 12B^2 + 40AB$$
$$y = 8A^2 - 24B^2 + 32AB$$
$$z = 2A^2 - 6B^2 + 36AB$$
$$w = 4A^2 + 12B^2$$

For the choice a=2A+1,b=2B+1

$$x = -4A^2 + 12B^2 + 16A + 32B + 40AB + 12$$
$$y = 8A^2 - 24B^2 + 24A - 8B + 32AB + 4$$
$$z = 2A^2 - 6B^2 + 20A + 12B + 36AB + 8$$
$$w = 4A^2 + 12B^2 + 4A + 12B + 4$$

Choice 4:
'1 ' can also be written as,

$$1 = \frac{(1 + i4\sqrt{3})(1 - i4\sqrt{3})}{49} \qquad (13)$$

Using (4),(5) & (13) in (10) and employing the method of factorization as in choice 1,the corresponding integer solutions are given by

$$x = \frac{1}{7}[-a^2 + 3b^2 - 74ab]$$
$$y = \frac{1}{7}[-19a^2 + 57b^2 - 34ab]$$
$$z = \frac{1}{7}[-10a^2 + 30b^2 - 54ab]$$

As our interest is of finding integer solutions, choose a and b suitably so that the solutions are in integers.
By taking a=7A, b=7B leads to the integer solutions to (1) to be,

$$x = -7A^2 + 21B^2 - 518AB$$
$$y = -133A^2 + 399B^2 - 238AB$$
$$z = -70A^2 + 210B^2 - 378AB$$
$$w = 49A^2 + 147B^2$$

Properties:

1) $19x(a, a^2 - 1) - y(a, a^2 - 1) + 4802(So)_a \equiv 0 \pmod{4802}$
2) $19x(7a - 5, a) - y(7a - 5, a) + 19208t_{9,a} = 0$
3) $w(1, b) + 7x(1, b) - t_{590,b} \equiv 0 \pmod{3333}$

Note:
In (13), '1' can also be written as

$$1 = \frac{(-1 + i4\sqrt{3})(-1 - i4\sqrt{3})}{49} \qquad (14)$$

For the choice , the integer solutions are

$$x = -133A^2 + 399B^2 + 238AB$$
$$y = -7A^2 + 21B^2 + 518AB$$
$$z = -70A^2 + 210B^2 + 378AB$$
$$w = 49A^2 + 147B^2$$

Choice 5:

Equation (3) can be re-written as

$$u^2 - 4w^2 = 3(w^2 - v^2)$$

which is written in the form of ratio as,

$$\frac{u+2w}{(w-v)} = \frac{3(w+v)}{u-2w} = \frac{a}{b} \qquad (15)$$

which is equivalent to the system of equations,

$$bu + av + (2b - a)w = 0$$
$$au - 3bv - (2a + 3b)w = 0$$

Applying the method of cross multiplication we have,

$$u = -2a^2 + 6b^2 - 6ab$$
$$v = -a^2 + 3b^2 + 4ab$$
$$w = a^2 + 3b^2$$

Substituting the values of 'u' and 'v' we get the non-zero distinct integral solutions to be

$$x = -3a^2 + 9b^2 - 2ab$$
$$y = -a^2 + 3b^2 - 10ab$$
$$z = -2a^2 + 6b^2 - 6ab$$

Properties:

1) $2w(2a, a^2) + z(2a, a^2) - 12(t_{4,a^2} + CP_a^6) = 0$
2) $x(2b-1, b) - 3y(2b-1, b) - 28bGbo_b = 0$
3) $z(a^2, a) - 2y(a^2, a) - 14CP_a^6 = 0$

Choice 6:

(15) can also be written as,

$$\frac{u+2w}{3(w-v)} = \frac{(w+v)}{u-2w} = \frac{a}{b}$$

which is equivalent to the system of equations,

$$bu + 3av + (2b - 3a)w = 0$$
$$au - bv - (2a + b)w = 0$$

Applying the method of cross multiplication we have,

$u = -6a^2 + 2b^2 - 6ab$

$v = -3a^2 + b^2 + 4ab$

$w = -3a^2 - b^2$

In view of (2) , we get the non-zero distinct integer solutions to be

$x = -9a^2 + 3b^2 - 2ab$

$y = -3a^2 + b^2 - 10ab$

$z = -6a^2 + 2b^2 - 6ab$

Properties:

1) $y(a, a^2) + z(a, a^2) - 3t_{4,a} + 16CP_a^6 + 9a = 0$

2) $x(a+1, a) - 3w(a+1, a) - 6t_{4,a} + 2\,\mathrm{Pr}_a = 0$

3) $-10y(a,a)$ & $-8x(a,a)$ are perfect squares.

Now instead of (2) , writing the linear transformations as

$$x = u + v,\ y = u - v,\ z = 4u \qquad (16)$$

in (1) , it leads to

$$u^2 + 3v^2 = 28w^2 \qquad (17)$$

Solving (17) in different ways one obtains other different choices of integer solutions to (1) which are illustrated as below:

Choice 7:

Write '28' as $28 = (5 + i\sqrt{3})(5 - i\sqrt{3})$ (18)

Using (4) and (18) in (17) and proceeding as in choice 1, the corresponding integer solutions are given by

$x = 6a^2 - 18b^2 + 4ab$

$y = 4a^2 - 12b^2 - 16ab$

$z = 24a^2 - 72b^2 - 16ab$

$w = a^2 + 3b^2$

Properties:

1) $x(a, 2a-1) + 6w(a, 2a-1) - 12t_{4,a} - 4t_{6,a} = 0$

2) $6y(6b, b-1) - z(6b, b-1) + 80S_b \equiv 0 \pmod{80}$

3) $x[a+1, a(a+2)] + 6w[a+1, a(a+2)] - 12t_{4,a} - 24P_a^3 \equiv 12 \pmod{24}$

4) $6y[1, a(2a^2+1)] - z[1, a(2a^2+1)] + 240(OH)_a = 0$

5)

Note:

In (18), '28' can also be considered as

$28 = (-5 + i\sqrt{3})(-5 - i\sqrt{3})$

For this choice the corresponding non-zero distinct integer solutions are obtained as

$x = -4a^2 + 12b^2 - 16ab$

$y = -6a^2 + 18b^2 + 4ab$

$z = -20a^2 + 60b^2 - 24ab$

$w = a^2 + 3b^2$

Choice 8:
Consider '28' as

$$28 = (4 + i2\sqrt{3})(4 - i2\sqrt{3}) \qquad (19)$$

Using (4) and (19) in (17) and proceeding as in choice 1, the integer solutions are as follows

$x = 6a^2 - 18b^2 - 4ab$

$y = 2a^2 - 6b^2 - 20ab$

$z = 16a^2 - 48b^2 - 48ab$

$w = a^2 + 3b^2$

Properties:

1) $5x(a,1) - y(a,1) - 28t_{4,a} + 84 = 0$
2) $y(2b^2 - 1, b) - 2w(2b^2 - 1, b) + 20(SO)_b + 12\Pr_b \equiv 0 \pmod{12}$
3) $x[a(a+1), a+2] - 3y[a(a+1), a+2] - 336P_a^3 = 0$
4) $z(6a, a-1) + 16w(6a, a-1) + 48(Sa-1) - 1152t_{4,a} = 0$

Note :

In (19),' 28' can be written as

$$28 = (-4 + i2\sqrt{3})(-4 - i2\sqrt{3}) \qquad (20)$$

By choosing this , the non-zero distinct integer solutions are given by

$x = -2a^2 + 6b^2 - 20ab$

$y = -6a^2 + 18b^2 - 4ab$

$z = -16a^2 + 48b^2 - 48ab$

$w = a^2 + 3b^2$

Choice 9:
(17) can also be written as

$$u^2 + 3v^2 = 28w^2 * 1 \qquad (21)$$

Using (14), (19) and (11) in (21) and proceeding as in pattern 1, the corresponding integral solutions are given by

$x = 2a^2 - 6b^2 - 20ab$

$y = -4a^2 + 12b^2 - 16ab$

$z = -4a^2 + 12b^2 - 72ab$

$w = a^2 + 3b^2$

Properties :

1) $x[1, a(2a^2 - 1)] + 2w[1, a(2a^2 - 1)] + 20(SO)_a$ is a perfect square.

2) $y[6a(a-1),1]+4w[6a(a-1),1]+16(S_a-1)$ is a nasty number.

3) $y(b+1,b+2)+4w(b+1,b+2)-S_b-2t_{4,b}\equiv 9(\bmod 54)$

4) $y[a^2+a,(a+2)(a+3)]-z[a^2+a,(a+2)(a+3)]-1344Pt_a=0$

Note :

Using (4), (12) and (20) in (21), we have the following set of solutions satisfying (1).

$$x=-4a^2+12b^2+16ab$$

$$y=2a^2-6b^2+20ab$$

$$z=-4a^2+12b^2+72ab$$

$$w=a^2+3b^2$$

Choice 10:

Using (4), (13) and (19) in (21) and employing the method of factorization we have the distinct integer solutions are as follows ;

$$x=\frac{1}{7}[-2a^2+6b^2-148ab]$$

$$y=\frac{1}{7}[-38a^2+114b^2-68ab]$$

$$z=\frac{1}{7}[-80a^2+240b^2-432ab]$$

As our aim is to find integral solutions choose a and b suitably so that the solutions are integers.

Taking a=7A, b=7B

$$x=-14A^2+42B^2-1036AB$$

$$y=-266A^2+798B^2-476AB$$

$$z=-560A^2+1680B^2-3024AB$$

$$w=49A^2+147B^2$$

Properties :

1) $z(a^2+a,2a^2-1)-40x(a^2+a,2a^2-1)+38416\Pr_a-76832t_{4,a}\Pr_a=0$

2) $7x(b+1,b)+2w(b+1,b)-588t_{4,b}+7252\Pr_b=0$

3) $19x(a,2a^2+1)-y(a,2a^2+1)+57624(OH)_a=0$

4) $19x(a(a+1),a+2)-y(a(a+1),a+2)+115248P_a^3=0$

Note :

Using (4), (14) and (20) in (21), we have the distinct integral solutions are obtained below

$$x=-266A^2+798B^2+476AB$$

$$y=-14A^2+42B^2+1036AB$$

$$z=-560A^2+1680B^2+3024AB$$

$$w=49A^2+147B^2$$

Choice 11:

(17) can be rewritten as

$$u^2-25w^2=3(w^2-v^2) \tag{22}$$

which is written in the form of ratio as

$$\frac{u+5w}{w+v}=\frac{3(w-v)}{u-5w}=\frac{a}{b} \qquad (23)$$

which is equivalent to the system of equations,

$$bu-av+(5b-a)w=0$$
$$au+3bv-(3b+5a)w=0$$

Applying the method of cross multiplication we have,

$$u=5a^2-15b^2+6ab$$
$$v=-a^2+3b^2+10ab$$
$$w=a^2+3b^2$$

Substituting the values of u ,v in (16) we get the non-zero distinct integer solutions to (1) to be

$$x=4a^2-12b^2+16ab$$
$$y=6a^2-18b^2-4ab$$
$$z=20a^2-60b^2+24ab$$
$$w=3a^2+b^2$$

Properties:

1) Each of the following expressions represents the nasty number.

a) $3\{x(a,2a^2+1)+4w(a,2a^2+1)-48(OH)_a\}$

b) $6y(b^2+b,b)+z(b^2+b,b)+224P_b^5-2b^4$

c) $3\{x(a,a^2+a)+4w(a,a^2+a)-32P_a^5\}$

2) $6w(2b^2-1,b)-y(2b^2-1,b)-4(SO)_b$ is a perfect square.

Choice 12:

(23) can also be written as

$$\frac{u+5w}{3(w+v)}=\frac{(w-v)}{u-5w}=\frac{a}{b}$$

which is equivalent to the system of equations,

$$bu-3av+(5b-3a)w=0$$
$$au+bv-(5a+b)w=0$$

Applying the method of cross multiplication we have,

$$u=15a^2-5b^2+6ab$$
$$v=-3a^2+b^2+10ab$$
$$w=3a^2+b^2$$

In view of (16) we get the non-zero distinct integer solutions to (1) are

$$x=12^2-4b^2+16ab$$
$$y=18a^2-6b^2-4ab$$
$$z=60a^2-20b^2+24ab$$
$$w=3a^2+b^2$$

Properties:

1) Each of the following expressions represents a perfect square:

   a) $2y(a,a)z(a,a)$

   b) $x(a+1,a)+t_{4,a}-16\Pr_a+4$

2) $x(a,a)$ and $4w(a,a)$ represents nasty numbers.

III. Conclusion

One may search for other Choices of solutions and their corresponding properties.

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