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Application of algebra in Peterson Graph

G. Nirmala[*] M. Murugan[**]

[*]Head and Associate Professor P.G. & Research Department of Mathematics, K.N. Govt. Arts College for Women (Autonomous) Thanjavur – 613 007

[**] Research Scholar Department of Mathematics, Periyar Maniyammai University Vallam, Thanjavur Email: ramlaxmp@yahoo.co.in, Contact: 9994837955

Abstract- In this work basic concepts of algebraic Graph theory and its properties are reviewed and extended to related concepts, of incidence matrix in Graph and Incidence matrix in a Peterson graph and its properties.

Index Terms- Peterson graph – Incidence matrix – Rank of the incidence matrix for a Peterson graph.

I. Introduction

Algebraic graph theory can be viewed as an extension to graph theory in which algebraic methods are applied to problem about graphs. Graph theory has found many applications in engineering and science such as chemical electrical civil and mechanical engineering, architecture, Management and control, communication, operation Research and computer Science.

A graph is completely determined by Specifying either its adjacency structure or its incidence structure.

Computer is more adopted at manipulating numbers than at recognising picture. It is a standard practice to communicate the specification of a graph to a computer matrix form.

This paper deals with Peterson graph and its properties with Incidence matrix and finding the Rank of the incidence matrix in a Peterson graph.

Definition 1

The degree of a vertex V_i in a graph G is the number of lines incident with v_i and is denoted by deg (v_i)
A vertex of degree '0' is called an Isolated point.
A Vertex of degree '1' is called the pendent Vertex.

Definition 2

For any graph G we defined δ (G) = min (deg v /v$\in$V(G) }
Δ (G) = max (deg v /v$\in$V(G) }
If all the Vertices of G have the Same degree r than
δ (G) = Δ (G) = r
Then G is called a regular graph of degree r
A regular graph of degree 3 is called a Cubic graph

Definition 3

Peterson graph is a 3- regular graph of 10 vertices and 15 edges.

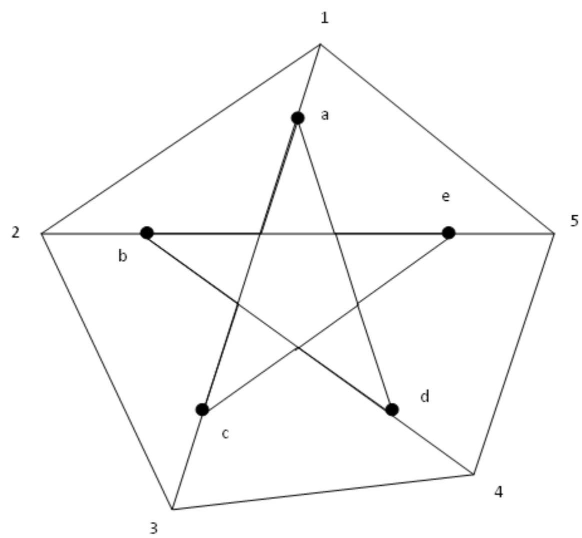

Figure 1

Peterson graph is a 3-regular graph

Definition 4

Incidence matrix, of a graph G With n vertices m edges and without self-loops is an n x m matrix A = (aij)

Whose n rows correspond to the 'n' vertices and the 'm' columns correspond to a m edges such that.

$$a_{ij} = \begin{cases} 1 & \text{if } j^{th} \text{ edge } m_j \text{ is incident on the } i^{th} \text{ vertex} \\ 0 & \text{otherwise} \end{cases}$$

It is also called vertex - edge incidence matrix and is denoted by A (G)

The incidence matrix contains only two types of elements 0 and 1.
It is a Binary matrix

Properties of the incidence matrix:

1. Every edge is incident on exactly two vertices. Each column of A has exactly two one's
2. The number of ones in each row equal the degree of the corresponding vertex.
3. A row with all Zeros represents Isolated Vertex.
4. Parallel edges in a graph produce identical columns in its incidence matrix.
5. The number of ones (or) number of Zeros are the same in every row in a incidence matrix Then the graph is regular.

6. If a graph G is disconnected and Consists of two components G_1 and G_2 The incidence matrix A(G) of a graph G can be written in a block diagonal form.

$$A(G) = \begin{bmatrix} A(G_1) & 0 \\ 0 & A(G_2) \end{bmatrix}$$

Where A (G_1) and A (G_2) are the incidence matrices of components G_1 and G_2

7. Permutation of any two rows (or) columns is an incidence matrix Simply corresponds to relabeling the vertices and edges of the same graph.
8. The incidence matrix A is defined over a field It is a Galois field module 2 It is denoted by GF (2)

Definition 5

Two graphs are said to be Isomorphic they have

(i) The same number of Vertices

(ii) The same number of edges

(iii) An equal number of vertices with a given degree

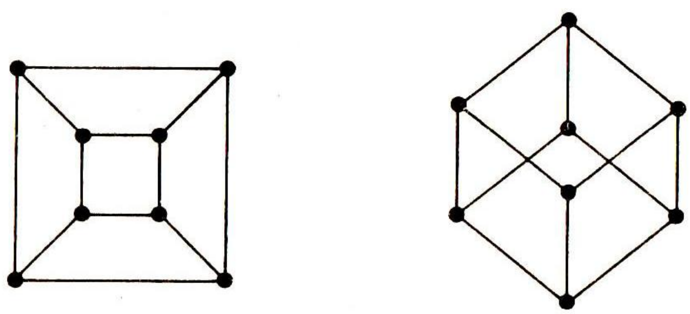

Figure 2: Isomorphic graph

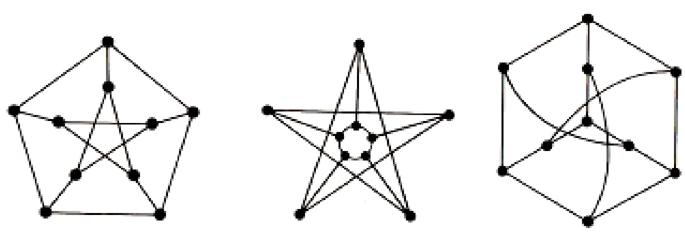

Figure 3 : Peterson graph . Isomorphic to some other graphs

Theorem 1

Two graphs are Isomorphic if and only if their incidence matrices differ only by permutation of rows and columns

Proof :

Let G_1 and G_2 are Isomorphic graphs and $I(G_1)$ & $I(G_2)$ are the incidence matrices

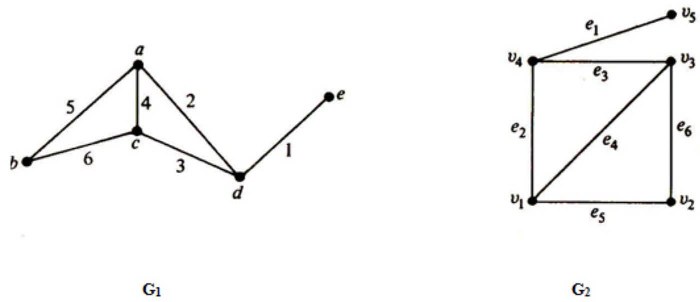

Figure 4

Incidence matrix of G_1 whose order 5 X 6.

$I(G_1) =$

	1	2	3	4	5	6
a	0	1	0	1	0	1
b	0	0	0	0	1	1
c	0	0	1	1	1	0
d	1	1	1	0	0	0
e	1	0	0	0	0	0

Incidence matrix of G_2 whose order 5 x 6

$I(G_2) =$

	E_1	E_2	E_3	E_4	E_5	E_6
V_1	0	1	0	1	1	0
V_2	0	0	0	0	1	1
V_3	0	0	1	1	0	1
V_4	1	1	1	0	0	0
V_5	1	0	0	0	0	0

It is clear that the incidence matrix of G_1 and G_2 differ only by the permutation of column 5 and column 6.

Let the graphs G_1 and G_2 be isomorphic Then there is a one-one correspondence between the vertices and edges in G_1 and G_2 such that the incidence relation is preserved.

Thus $I(G_1)$ and $I(G_2)$ are either same or differ only by permutation of rows and columns.

Conversely permutation of any two rows (or) columns in an incidence matrix simply corresponds to relabeling the vertices and edges of the same graph.

Hence the theorem

Rank of the incidence matrix:

Let G be a graph and A(G) be its incidence matrix. Now each row in A(G) is a vector over G_F (2) in the Vector space of a graph G . Let the row Vector be denoted by (A_1 A_2 ………….. A_n) Then

$$A(G) = \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ . \\ . \\ . \\ A_n \end{bmatrix}$$

Since they are exactly two ones in every column of A The sum of all these Vector is Zero.

$\therefore$ Then the Vectors A_1, A_2 ………… A_n are linearly dependent Therefore rank of A(G) <n
rank of A (G) $\leq$ n-1

Theorm (2)

If A (G) is an incidence matrix of a Connected graph G with n vertices then rank of A (G) is n -1

Proof:

Let G_1 be a connected graph with n=3.
Incidence matrix of G_1 is $A(G_1)$whose order is 3 x 2.

$$A(G_1) = \begin{matrix} & e_1 & e_2 \\ v_1 & 1 & 0 \\ v_2 & 1 & 1 \\ v_3 & 0 & 1 \end{matrix}_{3x2}$$

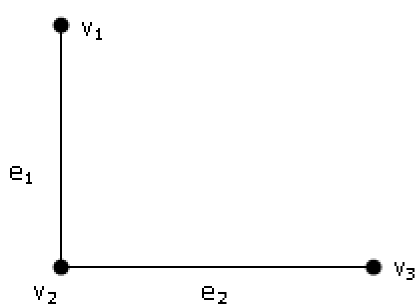

Figure 5: G_1

$$= \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \quad \& \quad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

Determinant of all possible minor matrices are not equal to zero.

$\Rightarrow$ Rank of $A(G_1)$ = 2. Similarly for n vertices.

$$A(G) = \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ . \\ . \\ . \\ A_n \end{bmatrix}$$

Clearly rank A (G) $\leq$ n-1 - (1)

Consider the sum of any m of these row Vector m $\leq$ n-1
Since G is connected A (G) cannot be partitioned in the form

$$A(G) = \begin{bmatrix} A(G_1) & 0 \\ 0 & A(G_2) \end{bmatrix}$$

Such that $A(G_1)$ has m rows and $A(G_2)$ has n-m rows
Then there exist a number of m x m sub matrix of A (G) for m$\leq$ n-1
Such that the modulo 2 sum of these m rows is equal to Zero.
As there are only two elements and 1 in this field the addition of all vectors taken m at a time for m= 1,2 ……….. n-1 gives all possible linear combination of n-1 row vector. Thus no linear combination of m row vector of A for m $\leq$ n-1 is Zero.
$\therefore$ Rank of A(G)$\geq$ n-1 ---- (2)

From (1) & (2) ; Rank of A (G) = n-1

Hence the theorem.

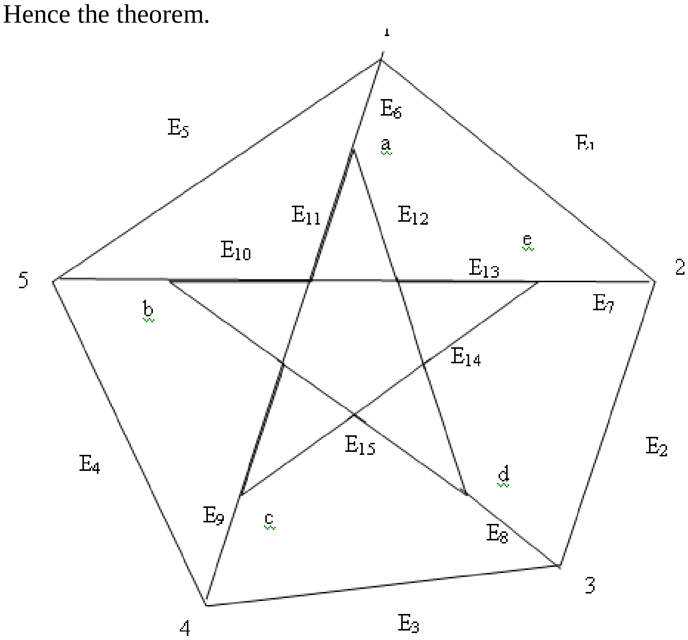

Figure 6

	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}
a	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0
b	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1
c	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0
d	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1
e	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0
1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
3	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0
4	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0
5	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0

10 x 15

II. Results

i) Every edges incident with exactly two vertices, then each column of the Incidence matrix of the Peterson graph has exactly two ones

ii) Every row has three ones (or) twelve Zero.
$\Rightarrow$ Peterson graph is 3-regular graph

iii) Sum of the each row is three it is equal to the degree of the corresponding vertex.

iv) The incidence matrix of a Peterson graph is defined over a field.

Galois field modulo 2 (or) GF(2) Such that the set (0,1) with operation addition modulo 2

0+0 = 0
1+0 =1
0+1 =1
1+1 =0

And multiplication modulo 2

0.0 =0
0.1=0
1.0 =0
1-1 =1

v) By theorem 2 : The rank of the incidence matrix of the Peterson graph is 9

Determinant value of the all sub matrix of order 10 x 10 is zero .

Determinant Value of the all sub matrix of order 9 x 9 is non-Zero.

Rank of the incidence matrix of the Peterson graph is 9.

III. Conclusion

To conclude that it is without doubt the Peterson graph is the generalization of the ordinary connected graph. From the nature of the Peterson graph many properties similar to those of ordinary connected graphs with the application of algebra can be extracted.

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AUTHORS

First Author – G. Nirmala, Head and Associate Professor P.G. & Research Department of Mathematics, K.N. Govt. Arts College for Women (Autonomous) Thanjavur – 613 007

Second Author – M. Murugan, Research Scholar Department of Mathematics, Periyar Maniyammai University Vallam, Thanjavur Email: ramlaxmp@yahoo.co.in, Contact: 9994837955