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Optimization of the Compressive Strength of Concrete Made With Aggregates from Oji-River Gravel Piths

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Abstract- Sedimentary rock aggregates are commonly used in Nigeria for structural concrete production. Due to its variability in quality, strength and contaminations it is being suspected as one of causes of many structural failures in the country. This paper investigated the quality of 8 – 16mm size gravels from Oji-River piths by optimizing the strength of concrete made from it through Scheffe's simplex method of optimization and produced a mathematical model which gave an optimum compressive strength of 11.78N/mm^2 and mix ratio of 1:1.4:2.5 for a water/cement ratio of 0.53. Because of the small nature of this optimum strength and the general predictions from the model, it was recommended that such aggregate (8 – 16mm from Oji-River) be restricted to oversite concretes, columns and lintels of buildings of load bearing walls and a maximum storey of three; and it should never be used for slabs of suspended floors, retaining walls, bridges and culverts.

I. INTRODUCTION

With the increasing rate of development in infra-structures in Nigeria sedimentary rock gravels are increasingly and commonly used for structural concrete production because of its cheapness and availability, when compared with granite. The problem with sedimentary rock aggregates is that they vary in quality over a wide range, especially strength and contaminations. For the fact that aggregates constitute over 70 percent of concrete volume, engineers are becoming more and more suspicious of sedimentary rock aggregates in connection with many structural failures that are experienced all over the country. In view of this, this paper wishes to investigate the quality of 8 – 16mm size gravel aggregates from Oji-River, a popular site for gavel aggregate collection in the South-Eastern part of Nigeria. The reason for the choice of 8 – 16mm size is that they are the worst contaminated with silt and clay due to method of production.

The method of optimization that will be used for the investigation is scheffe's simplex Lattice method for mixtures, where the property studied depends on the component ratios only. Firstly, a simplex is defined as a convex polyhedron with (k+1) vertices produced by k intersecting hyperplanes in k-dimensional space (Akhnazaova, 1982). Any co-ordinate system above 3-dimensions are referred to as hyper planes, such planes are not orthogonal. A 2-dimensional regular simplex is, therefore, an equilateral triangle, while a 3-dimensional regular simplex is a regular tetrahedron.

Scheffe (1958) used a regular (q – 1) – simplex to represent a factor space needed to describe a response surface for mixtures consisting of several components. If the number of components is denoted by q, then for binary system (q = 2) the required simplex is a straight line; for q = 3, the required simplex is an equilateral triangle; and for q = 4, the simplex is a regular tetrahedron. The response surface for such a multicomponent system is normally described by means of a high degree polynomial, of the type of Eq 1.0, having number of coefficients given by C^n_{q+n} , where n is the degree of the polynomial (Akhanzarova et al, 1982).

$$\hat{y} = b_o + \sum_{1\le i\le q} b_i x_i + \sum_{1\le i<j\le q} b_{ij} x_i x_j + \sum_{1\le i<j<k\le q} b_{ijk} x_i x_j x_k + \sum bi_1 i_2 \dots i_n xi_1 xi_2 \dots xi_n \qquad (1.0)$$

Knowing that Eq (2.0) also holds for mixtures

$$\sum_{i=1}^{q} x_i = 1 \qquad - - - \quad (2.0)$$

Where $x_i \ge 0$ represents the component concentrations in the mixture, Scheffe (1958) was able to reduce the number of coefficients in Eq (1.0) to arrive at a new polynomial whose number of coefficients is given by C^n_{q+n-1}, thereby reducing the number of experimental trials required to evaluate the coefficients. Scheffe's reduced polynomial is more preferred and commonly used. To demonstrate this reduction for a four-component mixture, we have:

From Eq (1.0) and Eq (2.0)

$$\hat{y} = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + +b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 \quad - - - - - - \quad (3.0)$$

$$and\ x_1 + x_2 + x_3 + x_4 = 1 \quad - - - - \quad (4.0)$$

Multiplying Eq (4.0) by b_o, x_1, x_2, x_3 and x_4, separately, and rearranging the variables the following equations are obtained;

$$b_o = b_ox_1 + b_ox_2 + b_ox_3 + b_ox_4 \quad - - - \quad (5.0)$$
$$x_1^2 = x_1 - x_1x_2 - x_1x_3 - x_1x_4 \quad - - - - \quad (6.0)$$
$$x_2^2 = x_2 - x_1x_2 - x_2x_3 - x_2x_4 \quad - - - - \quad (7.0)$$
$$x_3^2 = x_3 - x_1x_3 - x_2x_3 - x_3x_4 \quad - - - - \quad (8.0)$$
$$x_4^2 = x_4 - x_1x_4 - x_2x_4 - x_3x_4 \quad - - - - \quad (9.0)$$

Substituting Eqs 5.0, 6.0, 7.0, 8.0 and 9.0 into Eq. 3.0 and rearranging, yields

$$\hat{Y} = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_{12}x_1x_2 \quad \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{34}x_3x_4 \quad - - - - - \quad -(10)$$

Eq(10) is the scheffe's reduced second degree polynomial for 4-component mixtures. It has only 10 coefficients instead of 15, reducing the number of experimental trials by 5.

1.2 Factor Notation on a Simplex Lattice

Each component to be used in a mixture is divided into (n+1) similar level (parts), where n is the degree of the polynomial to be used in the model. The component compositions and their respective concentrations in each mixture are shown by the use of subscripts. For example, a mixture x_{ij} could contain only one component with its full concentration denoted as x_1, x_2, x_3 or x_4 ; another mixture could contain two components of equal concentrations denoted as x_{12}, x_{13}, x_{14}, x_{23}, x_{24} or x_{34}. A mixture having two components of different concentrations is denoted as x_{112}, x_{113}, x_{224}, etc. – the number of times each of the components appears in the subscript relative to the other is a measure of their relative concentration.

These mixtures are arrayed on the simplex to form a lattice, i.e. a uniform scatter that could be joined by crossing straight lines parallel to the edges of the simplex. For tetrahedrons, for instance, starting from the vertex with straight component mixtures x_1, x_2, x_3, etc; followed by the edges with binary component mixtures x_{12}, x_{13}, x_{24}, etc; then the faces with 3-component mixture x_{124}, x_{234} etc; and finally the interior with 4-component mixtures, this sequence is followed until all the required experimental trials are depicted on the simplex. Fig. 1.0 shows the positions of all the factors (mixtures) on a regular tetrahedron for a second degree polynomial to be used for the description of the response space for a 4-component mixture – a (4, 2) – lattice.

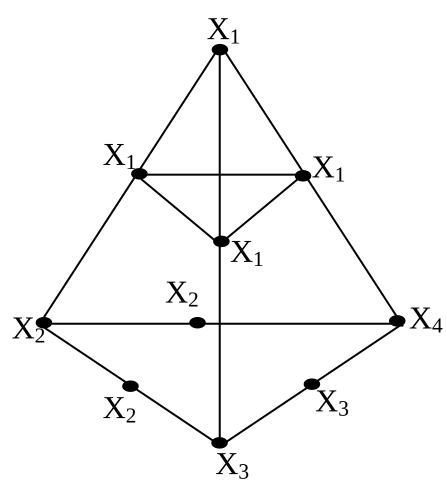

Fig. 1.0: Factor Notations for a (4, 2) – Lattice

A matrix table is normally used to display these factors (see leftside of table 1.0) each row displaying a mixture with its components and concentrations.

Table 1.0: Matrix table for Scheffe's (4, 2) – Lattice Polynomial

Pseudo-Components					Response	Real Components			
S/N	x_1	X_2	X_3	X_4		Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.6	1.0	1.5	4
2	0	1	0	0	Y_2	0.5	1.0	1.0	1½
3	0	0	1	0	Y_3	0.55	1.0	1½	3.0
4	0	0	0	1	Y_4	0.555	1.0	$2\,{}^{1}/_{2}$	4.0
5	$^{1}/_{2}$	$^{1}/_{2}$	0	0	Y_{12}	0.55	1.0	1.25	2.75
6	$^{1}/_{2}$	0	$^{1}/_{2}$	0	Y_{13}	0.575	1.0	1.5	3.5
7	$^{1}/_{2}$	0	0	$^{1}/_{2}$	Y_{14}	0.578	1.0	2.0	4.0
8	0	$^{1}/_{2}$	$^{1}/_{2}$	0	Y_{23}	0.525	1.0	1.25	2.25
9	0	$^{1}/_{2}$	0	$^{1}/_{2}$	Y_{24}	0.528	1.0	1.75	2.75
10	0	0	$^{1}/_{2}$	$^{1}/_{2}$	Y_{34}	0.553	1.0	2.0	3.5

For the fact that concrete mixtures have its sum of proportions above unity a congruent simplex is necessary such that the mix proportions at the vertices show the range of w/c ratio, cement, fine aggregate and coarse aggregate ratios, respectively, the required polynomial model will cover or predict (see fig. 2.0).

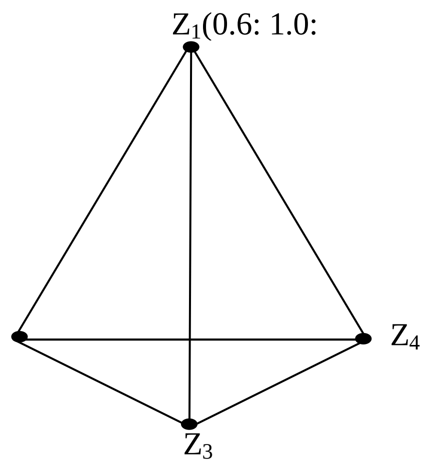

Fig. 2.0: Real Component Simplex (only vertices are shown)

The former simplex, fig. 1.0, is called Pseudo-component simplex and the later, fig. 2.0, real component simplex. From the later (real components) a Z-matrix is formed whose transpose becomes the conversion factor from pseudo to real component; i.e. from fig. 2.0

$$z = \begin{bmatrix} 0.6 & 1.0 & 1.5 & 4.0 \\ 0.5 & 1.0 & 1.0 & 1\frac{1}{2} \\ 0.55 & 1.0 & 1\frac{1}{2} & 3.0 \\ 0.555 & 1.0 & 2\frac{1}{2} & 4.0 \end{bmatrix}$$

$$and\ Z^T = \begin{bmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1\frac{1}{2} & 2\frac{1}{2} \\ 4.0 & 1\frac{1}{2} & 3.0 & 4.0 \end{bmatrix} \quad - \quad - \quad - \quad 11$$

To demonstrate the use of Eq (11) in table 1.0, the 5th row in the real component side is obtained by multiplying $[Z]^T$ matrix by the corresponding row in the pseudo-component side of table 1.0, i.e.

$$= \begin{bmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1\frac{1}{2} & 2\frac{1}{2} \\ 4.0 & 1\frac{1}{2} & 3.0 & 4.0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.55 \\ 1.0 \\ 1.25 \\ 2.75 \end{bmatrix}$$

In this way all the corresponding rows in the real component side are obtained producing a congruent table and simplex suitable for concrete.

II. MATERIAL AND METHOD

(i) Materials

Materials needed for the experiment include sample of unwashed coarse aggregate (gravel) from Oji-River gravel pith. The samples were stored in sacks, indoor, so that moisture variation in the samples would be minimal.

The laboratory equipments needed include, universal crushing machine, 150 x 150mm x 150mm cube moulds, mould oil, weighing balance, trowel and curing tank.

(ii) Method

Using the weighing balance; water, cement fine aggregate and coarse aggregate were weighed out, respectively, in the

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proportions shown in table 1.0- right side - in such a way that the materials weighed out served for three cubes. The materials were thoroughly mixed together inside a non-absorbent container before water was added and final mixing was done. Three cubes were cast from each of the mix proportions, making 60 cubes in the whole. The fresh concrete was filled into the moulds in three layers, each layer tamped not less than 25 times. The top was scraped off with the trowel. The concrete was allowed to harden for 24 hours, after which the mould was removed and the cubes were water-cured for 28 days in the curing tank. At the end of 28 days the cubes were crushed in the universal crushing machine. The results and averages from the test points were tabulated in columns 7 to 10 of table 2.0. Extra ten test points were provided for validation of the model. The number of extra test point depends on choice.

Table 2.0: Responses from Experiment and Predictions from Model

S/N	Pseudo-Components				Response Symbols	Replicate Responses (N/mm^2)			Average Response $\bar{Y} N/mm^2$	Predicted values $\hat{Y} N/mm^2$	Real Components (Concrete Mix ratios)			
	X_1	X_2	X_3	X_4		1	2	3						
1	1	0	0	0	Y_1	6.89	6.44	8.22	7.18	7.18	0.6	1	1 ½	4
2	0	1	0	0	Y_2	7.78	7.11	6.89	7.26	7.26	0.5	1	1	1 ½
3	0	0	1	0	Y_3	9.78	8.0	8.0	8.59	8.59	0.55	1	1 ½	3
4	0	0	0	1	Y_4	4.44	5.33	7.33	5.7	5.7	0.555	1	2 ½	4
5	$1/2$	$1/2$	0	0	Y_{12}	7.55	9.33	8.44	8.44	8.44	0.55	1	1¼	2¾
6	$1/2$	0	$1/2$	0	Y_{13}	8.44	7.78	10.22	8.81	8.81	0.575	1	1½	3 ½
7	$1/2$	0	0	$1/2$	Y_{14}	9.55	8.44	10.00	9.33	9.33	0.578	1	2	4
8	0	$1/2$	$1/2$	0	Y_{23}	13.78	11.11	10.00	11.63	11.63	0.525	1	1 ¼	2¼
9	0	$1/2$	0	$1/2$	Y_{24}	10.00	10.00	10.00	10.00	10.00	0.528	1	1¾	2¾
10	0	0	$1/2$	$1/2$	Y_{34}	10.67	8.67	9.33	9.56	9.56	0.533	1	2	3½
	Control Points													
11	$1/2$	0	$1/4$	$1/4$	C_1	10.94	8.67	9.56	9.56	9.82	0.576	1	2 ¾	3 ¾
12	$1/4$	0	$1/2$	$1/4$	C_2	11.11	8.22	10.00	9.78	9.98	0.564	1	1¾	3½
13	$1/4$	$1/4$	$1/4$	$1/4$	C_3	11.00	8.11	8.89	9.33	10.92	0.551	1	1.625	3.125
14	$2/3$	0	0	$1/3$	C_4	8.44	11.11	7.11	8.89	9.52	0.585	1	1.833	4.0
15	$1/4$	$1/4$	$1/2$	0	C_5	9.56	10.44	11.56	10.52	10.53	0.55	1	1.375	2.875
16	$1/4$	$1/2$	0	$1/4$	C_6	7.56	11.56	14.11	10.74	10.02	0.539	1	1 ½	2 ¾
17	$1/4$	0	$1/4$	$1/2$	C_7	5.79	8.89	8.22	7.63	9.82	0.535	1	2	3 ¾
18	$1/2$	$1/4$	0	$1/4$	C_8	6.11	9.56	9.89	8.52	9.91	0.564	1	1.625	3.375
19	$1/4$	$1/2$	$1/8$	$1/8$	C_9	10.89	8.67	10.22	9.93	10.29	0.538	1	1.375	2.625
20	$1/3$	$1/3$	0	$1/3$	C_{10}	9.76	10.42	7.67	9.28	10.24	0.552	1	1.667	3.167

2.1Development of the Model

The general form of Scheffe's (4,2) – Lattice Polynomial is given by

$$\hat{Y} = \sum_{1\le i\le 4} \beta_i x_i + \sum_{1\le i<j\le 4} \beta_{ij} x_i x_j - \quad - \quad - \quad - \quad 12$$

$$where\ \beta_i = \bar{y}_i\ , \beta_{ij} = 4\bar{y}_{ij} - 2\bar{y}_i - 2\bar{y}_j$$

From table 2.0, Column 10:

$$\beta_1 = 7.18, \beta_2 = 7.26, \beta_3 = 8.59, \beta_4 = 5.7$$

$$\beta_{12} = 4 \times 8.44 - 2 \times 7.18 - 2 \times 7.26 = 4.88$$

$$\beta_{13} = 4 \times 8.81 - 2 \times 7.18 - 2 \times 8.59 = 3.70$$

$$\beta_{14} = 4 \times 9.33 - 2 \times 7.18 - 2 \times 5.7 = 12.76$$

$$\beta_{23} = 4 \times 11.63 - 2 \times 7.26 - 2 \times 8.59 = 14.82$$

$$\beta_{24} = 4 \times 10.00 - 2 \times 7.26 - 2 \times 5.7 = 14.08$$
$$\beta_{34} = 4 \times 9.56 - 2 \times 8.59 - 2 \times 5.7 = 9.66$$

The model for compressive strength for Oji-River sample becomes

$$\hat{Y} = 7.18x_1 + 7.26x_2 + 8.59x_3 + 5.7x_4 + 4.88x_1x_2 + 3.70x_1x_3 + 12.76x_1x_4 + 14.82x_2x_3 + 14.08x_2x_4 + 9.66x_3x_4 \quad - \quad - \quad - \quad - \quad 13$$

The predictions from Eq 13 are given in table 2.0 Column 11.

2.2 Validation of the Model (Test for Adequacy)

Adequacy of the model (Eq 13) can be tested through Fisher's variance ratio, whereby the calculated value of Fisher's ratio F is compared with the tabulated value in the Quantile of the F-Distribution.

$$F = \frac{S_g^2}{S_e^2} \quad - \quad - \quad - \quad - \quad - \quad 14$$

$$Where\ Sg^2 = \frac{m}{n-l}\sum_{i=1}^{n}(\bar{y}_i - \hat{y}_i)^2 - \quad - \quad - 15$$

$$and\ S_e^2 = \frac{1}{n(m-1)}\sum_{i=1}^{n}\sum_{u=1}^{n}(y_i - \bar{y}_i)^2 \quad - \quad - \quad - \quad - \quad 16$$

In the above equations n is the number of experimental trials, m is the number of replications for each experimental trial, *l* is the number of coefficients in the model, $\bar{y}_i$ is the average response for i^{th} experimental trial, $\hat{y}_i$ is the predicted value from model for i^{th} trial, y_{iu} is the u^{th} replicate response value for i^{th} trial. If F is less than the tabulated value, then the model is adequate, i.e.

$$S_g^2 = \frac{3}{10} \times 11.3305$$
$$S_e^2 = \frac{1}{40} \times 82.3513$$
$$F = \frac{S_g^2}{S_e^2} = \frac{(3/10) \times 113305}{(1/4) \times 82.3513}$$
$$= 1.65 < 2.1 \quad okay$$

Where the value 2.1 is the limiting value of F obtained from any table of Quantiles of F-Distribution.

2.3 Optimization of the Model

The model (Eq 13) was optimized through a Quick-Basic computer program, whose flowchart is given in Fig. 3.0. The maximum values given by the computer for strength, water, cement, fine aggregate and coarse aggregate ratios are 11.78KN, 0.531, 1, 1.4 and 2.5 respectively.

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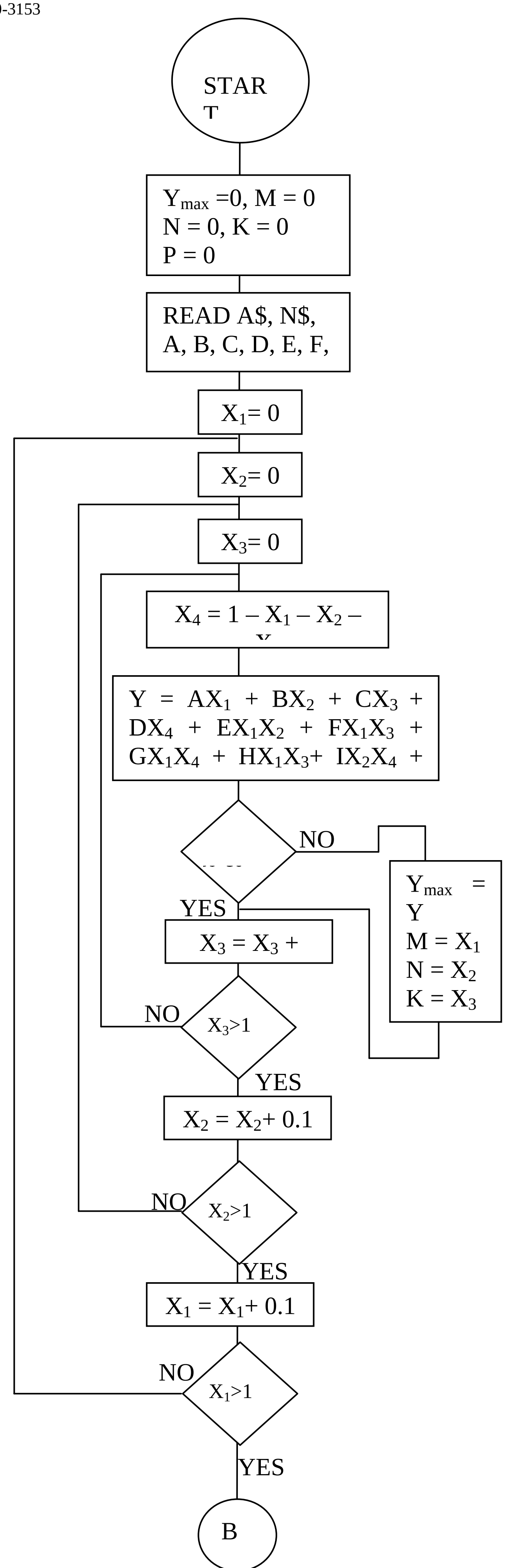


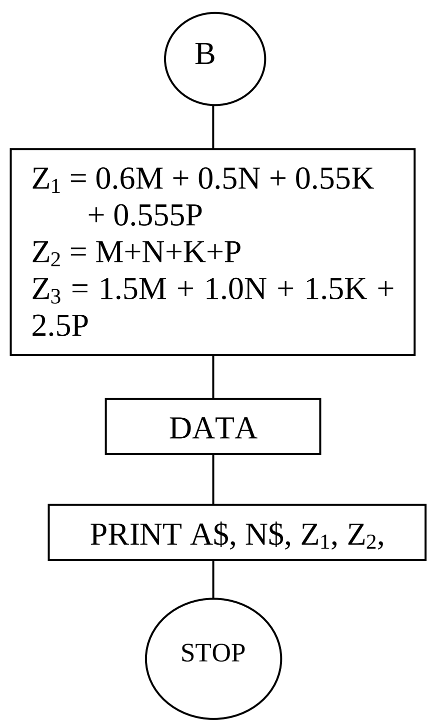
KEY

A$ = Gravel Pith

N$ = Strength Type

A,B,C - - J are the coefficients of the model

Ymax, Z_1, Z_2, Z_3 and Z_4 are the maximum strength, water, cement, fine agg, coarse agg ratios respectively



2.4 Discussion of Results

Looking at the results and predictions from the model in table 2.0 (Columns 7, 8, 9, 10 and 11) it is easy to see that the compressive cube strengths for the various mix proportions are clearly below the expected values by a wide margin. Considering the optimum value given by the computer program, whose proportions are comparable with that of grade 25 concrete, when granite is used instead - it has an average strength of 11.78N/mm^2, which is about half of the expected value - this shows that aggregates of sizes 8 – 16mm from Oji-River are quite inferior.

III. Recommendation and Conclusion

From the above results and discussions it is obvious that concretes from these aggregates cannot be used in areas where there is excessive compressive and tensile forces such as bridges, culverts, thin slabs, foundation, etc. It is therefore recommended for only columns and lintel of load bearing walls, the storey should not be greater than three.

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