

# Fuzzy rw Super- Continuous Mapping

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**Abstract-** In this paper we extend the concepts of rw super closed sets and rw super continuous mappings in fuzzy topological spaces and obtain several results concerning the preservation of fuzzy g- super closed sets. Furthermore we characterize fuzzy rw super continuous and fuzzy rw- super closed mappings and obtain some of the basic properties and characterization of these mappings.

**Index Terms-** Fuzzy super closure fuzzy super interior fuzzy super closed set, fuzzy super open set fuzzy g- super closed sets, fuzzy g- super open sets, fuzzy g- super continuous, fuzzy rw super closed, fuzzy rw -super continuous and fuzzy gc-super irresolute mappings.

## I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [17] in 1965 and fuzzy topology by Chang [3] in 1968, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Thakur and Malviya [14] introduced the concepts of fuzzy g- closed sets, fuzzy g-continuity and fuzzy gc-irresolute mappings in fuzzy topological spaces.

In this paper we introduce the concepts of fuzzy rw -super closed and fuzzy rw -super continuous mappings using fuzzy g- super closed sets. This definition enables us to obtain conditions under which maps and inverse maps preserve fuzzy g- super closed sets. We also characterize fuzzy  $T_{1/2}$ -spaces in terms of fuzzy rw super continuous and fuzzy rw -super closed mappings. Finally some of the basic properties of fuzzy rw super continuous and fuzzy a- super closed mappings are investigated.

## II. PRELIMINARIES

Let  $X$  be a non empty set and  $I = [0,1]$ . A fuzzy set on  $X$  is a mapping from  $X$  into  $I$ . The null fuzzy set  $0$  is the mapping from  $X$  into  $I$  which assumes only the value is  $0$  and whole fuzzy sets  $1$  is a mapping from  $X$  onto  $I$  which takes the values  $1$  only. The union (resp. intersection) of a family  $\{A_\alpha: \alpha \in \Lambda\}$  of fuzzy sets of  $X$  is defined by to be the mapping  $\sup A_\alpha$  (resp.  $\inf A_\alpha$ ). A fuzzy set  $A$  of  $X$  is contained in a fuzzy set  $B$  of  $X$  if  $A(x) \leq B(x)$  for each  $x \in X$ . A fuzzy point  $x_\beta$  in  $X$  is a fuzzy set defined by  $x_\beta(y) = \beta$  for  $y = x$  and  $x_\beta(y) = 0$  for  $y \neq x, \beta \in [0,1]$  and  $y \in X$ . A fuzzy point  $x_\beta$  is said to be quasi-coincident with the fuzzy set  $A$  denoted by  $x_\beta qA$  if and only if  $\beta + A(x) > 1$ . A fuzzy set  $A$  is quasi-coincident with a fuzzy set  $B$  denoted by  $A_qB$  if and only if there exists a point  $x \in X$  such that  $A(x) + B(x) > 1$ . For any two fuzzy sets  $A$  and  $B$  of  $X$ ,  $A \leq B$  if and only if  $\overline{(A_qB^c)}$  [5]. A family  $\tau$  of fuzzy sets of  $X$  is called a fuzzy topology [1] on  $X$  if  $0,1$  belongs to  $\tau$  and  $\tau$  is closed with respect to arbitrary union and finite intersection. The members of  $\tau$  are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set  $A$  of  $X$  the closure of  $A$  (denoted by  $cl(A)$ ) is the intersection of all the fuzzy closed sets of  $A$  and the interior of  $A$  (denoted by  $int(A)$ ) is the union of all fuzzy open subsets of  $A$ .

**Definition 2.1 [6]:** Let  $(X, \tau)$  fuzzy topological space and  $A \subseteq X$  then

1. Fuzzy Super closure  $scl(A) = \{x \in X: cl(U) \cap A \neq \emptyset\}$
2. Fuzzy Super interior  $sint(A) = \{x \in X: cl(U) \leq A \neq \emptyset\}$

**Definition 2.2[5,6]:** A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called:

- (a) Fuzzy super closed if  $scl(A) \leq A$ .
- (b) Fuzzy super open if  $1-A$  is fuzzy super closed  $scl(A) = A$

**Remark 2.1[5,6]:** Every fuzzy closed set is fuzzy super closed but the converses may not be true.

**Remark 2.2[5,6]:** The intersection of two fuzzy super closed sets in a fuzzy topological space  $(X, \mathfrak{S})$  may not be fuzzy super closed.  
For

**Definition 2.2[1,5,6,7]:** A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called:

- (a) fuzzy semi super open if there exists a super open set  $O$  such that  $O \leq A \leq \text{cl}(O)$ .
- (b) fuzzy semi super closed if its complement  $1-A$  is fuzzy semi super open.

**Remark 2.3[1,5,7]:** Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true .

**Definition 2.3[7]:** A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called fuzzy  $w$ -super closed if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy semi super open.

**Remark 2.4[5,6]:** Every fuzzy super closed set is fuzzy  $w$ -super closed but its converse may not be true. For,

**Definition 2.4[3,7]:** A fuzzy sets  $A$  of a fuzzy topological spaces  $(X, \mathfrak{S})$  is called fuzzy regular super open if  $A = \text{int}(\text{cl}(A))$ .

**Definition 2.5[3,7]:** A fuzzy sets  $A$  of a fuzzy topological spaces  $(X, \mathfrak{S})$  is called fuzzy regular super closed if  $A = \text{cl}(\text{int}(A))$  .

**Remark 2.5:** Every fuzzy open (resp. fuzzy regular super closed) set is fuzzy regular super open (resp. fuzzy regular super closed) but the converse may not be true [ ].The family of all fuzzy regular super open (resp. fuzzy regular super closed) sets of a fuzzy topological  $(X, \mathfrak{S})$  will be denoted by  $\text{FRO}(X)$ ( resp.  $\text{FRC}(X)$ ).

**DEFINITION 2.6[3,5,6,7]:** A mapping  $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$  is said to be fuzzy almost super continuous if  $f^{-1}(G) \in \mathfrak{S}$  for each fuzzy set of  $G \in \text{FRO}(Y)$  .

**Remark 2.6[6]:** Every fuzzy super continuous mapping is fuzzy almost super continuous but the converse may not be true [7].

**Definition 2.7:** A fuzzy sets  $A$  of a fuzzy topological spaces  $(X, \mathfrak{S})$  is called fuzzy regular semi super open if there exists a fuzzy regular super open set  $O$  such that  $O \leq A \leq \text{cl}(O)$  [6]

The family of all fuzzy regular semi super open sets of a fuzzy topological  $(X, \tau)$  will be denoted by  $\text{FRSSO}(X)$ .

**Remark 2.7:** Every fuzzy regular super open set is fuzzy regular semi super open but the converse may not be true .

**Definition 2.8:** A mapping  $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$  is said to be fuzzy almost super irresolute if the inverse image of every fuzzy regular semi super open set of  $Y$  is fuzzy semi super open in  $X$ . [6]

**Remark 2.8:** Every fuzzy super irresolute mapping is fuzzy almost super irresolute but the converse may not be true [ $P_6$ ].

**Definition 2.9:** A fuzzy set  $A$  of a topological spaces  $(X, \mathfrak{S})$  is called fuzzy  $rg$ - super closed if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy regular super open in  $X$  .

**Remark 2.9:** Every fuzzy  $g$ - super closed set is fuzzy  $rg$ - super closed but its converse may not be true.

**Definition 2.10:** A mapping  $f: (X, \mathfrak{S}) \rightarrow (Y, \Gamma)$  is said to be fuzzy  $rg$ -super continuous if the inverse image of every fuzzy super closed set of  $Y$  is fuzzy  $rg$ - super closed set in  $X$ .

**Remark 2.10:** Every fuzzy  $g$ -super continuous mapping is fuzzy  $rg$ -super continuous but the converse may not be true .

### III. FUZZY RW SUPER-CLOSED SETS

In the present section we introduce the concepts of fuzzy  $rw$  super-closed sets in fuzzy topology and obtained some of its basic properties.

**Definition 3.1:** A fuzzy set  $A$  of a topological spaces  $(X, \mathfrak{S})$  is called fuzzy  $rw$  super-closed if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy regular semi super open in  $X$ .

**Remark 3.1:** Every fuzzy  $w$ - super closed set is fuzzy  $rw$  - super closed but its converse may not be true for,

**Example 3.1:** Let  $X = \{a, b\}$  and the fuzzy sets  $A$  and  $U$  be defined as follows:

$$A(a)=0.7, A(b)=0.8, U(a)=0.7, U(b)=0.6$$

Let  $\tau = \{0, U, 1\}$  be a fuzzy topology on  $X$ . Then  $A$  is fuzzy rw- super closed but not fuzzy w- super closed.

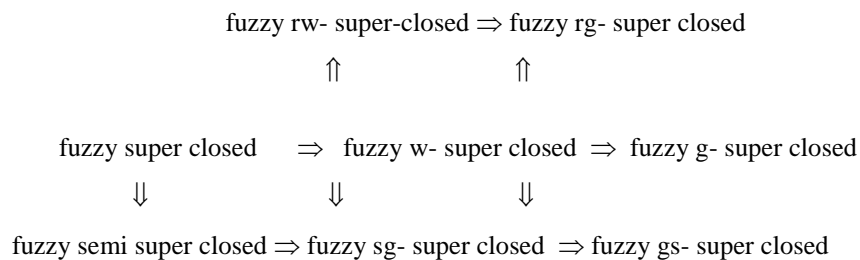
**Remark 3.2:** Every fuzzy rw- super-closed set is fuzzy rg- super closed but not conversely. For,

**Example 3.2:** Let  $X = \{a, b, c, d\}$  and the fuzzy sets  $O, U, V, W,$  and  $A$  are defined as follows:

$$O(a) = 1, O(b) = 0, O(c) = 0, O(d) = 0, U(a) = 0, U(b) = 1, U(c) = 0, U(d) = 0$$

$$V(a)=1, V(b) = 1, V(c) = 0, V(d) = 0, W(a) = 0, W(b) = 0, W(c) = 1, W(d) = 1$$

$A(a) = 0, A(b) = 0, A(c) = 1, A(d) = 0$ , Let  $\tau = \{0, O, U, V, W, 1\}$  be the fuzzy topology on  $X$ . then  $A$  is rw super-closed but not rg- super closed. Thus we have the following diagram of implications:



**Theorem 3.1:** Let  $(X, \mathfrak{T})$  be a fuzzy topological spaces and  $A$  is fuzzy subset of  $X$ . Then  $A$  is fuzzy rw super-closed if and only if  $\overline{\text{cl}}(A_q F) \Rightarrow \overline{\text{cl}}(\text{cl}(A)_q F)$  for every fuzzy regular semi super closed set  $F$  of  $X$ .

**Proof: Necessity:** Let  $F$  be a fuzzy regular semi super closed subsets of  $X$  and  $\overline{\text{cl}}(A_q F)$ . Then  $A \leq 1-F$  and  $1-F$  is fuzzy regular semi super open in  $X$ . Therefore  $\text{cl}(A) \leq 1-F$  because  $A$  is fuzzy rw -super closed. Hence  $\overline{\text{cl}}(\text{cl}(A)_q F)$ .

**Sufficiency:** Let  $U \in \text{FRSSO}(X)$  such that  $A \leq U$  then  $\overline{\text{cl}}(A_q(1-U))$  and  $1-U$  is fuzzy regular semi super closed in  $X$ . Hence by hypothesis  $\overline{\text{cl}}(\text{cl}(A)_q(1-U))$ . Therefore  $\text{cl}(A) \leq U$ , Hence  $A$  is fuzzy rw super closed in  $X$ .

**Theorem 3.2:** Let  $A$  be a fuzzy rw- super closed set in a fuzzy topological space  $(X, \tau)$  and  $x_\beta$  be a fuzzy point of  $X$  such that  $x_\beta q(\text{cl}(\text{int}(A)))$  then  $\text{cl}(\text{int}(x_\beta))_q A$ .

**Proof:** If  $\overline{\text{cl}}(\text{cl}(\text{int}(x_\beta))_q A)$  then  $A \leq 1-\text{cl}(\text{int}(x_\beta))$  and so  $\text{cl}(A) \leq 1-\text{cl}(\text{int}(x_\beta)) \leq 1-x_\beta$  because  $A$  is fuzzy rw super closed in  $X$ . Hence  $\overline{\text{cl}}(x_\beta q \text{cl}(\text{int}(A)))$ , a contradiction.

**Theorem 3.3:** If  $A$  and  $B$  are fuzzy rw- super closed sets in a fuzzy topological space  $(X, \tau)$  then  $A \cup B$  is fuzzy rg- super closed.

**Proof:** Let  $U \in \text{FRSO}(X)$  such that  $A \cup B \leq U$ . Then  $A \leq U$  and  $B \leq U$ , so  $\text{cl}(A) \leq U$  and  $\text{cl}(B) \leq U$ . Therefore  $\text{cl}(A) \cup \text{cl}(B) \leq \text{cl}(A \cup B) \leq U$ . Hence  $A \cup B$  is fuzzy rw-super closed.

**Remark 3.3:** The intersection of any two fuzzy rw -super closed sets in a fuzzy topological space  $(X, \tau)$  may not be fuzzy rw -super closed for,

**Example 3.3:** Let  $X = \{a, b, c, d\}$  and the fuzzy sets  $A$  and  $B$  are defined as follows;

$$A(a)=1, \quad A(b)=1, \quad A(c)=0, \quad A(d)=0,$$

$$B(a)=1, \quad B(b)=0, \quad B(c)=1, \quad B(d)=1,$$

Let  $\mathfrak{T} = \{0, A, B, A \cap B, 1\}$  be the fuzzy topology on  $X$ . Then  $A$  and  $B$  are fuzzy rw -super closed but their intersection  $A \cap B$  is not fuzzy rw -super closed.

**Theorem 3.4:** Let  $A \leq B \leq \text{cl}(A)$  and  $A$  is fuzzy rw -super closed set in a fuzzy topological space  $(X, \tau)$  then  $B$  is fuzzy rw -super closed.

**Proof:** Let  $U \in \text{FRSO}(X)$  such that  $B \leq U$ . Then  $A \leq U$  and since  $A$  is fuzzy rw- super closed. Then  $\text{cl}(A) \leq U$ . Now  $B \leq \text{cl}(A) \Rightarrow \text{cl}(B) \leq \text{cl}(A) \leq U$ . Consequently  $B$  is fuzzy rw -super closed.

**Definition 3.2:** A fuzzy set  $A$  of a fuzzy topological space  $(X, \mathfrak{T})$  is called fuzzy rw -super open) if and only if  $1 - A$  is fuzzy rw -super closed.

**Remark 3.4:** Every fuzzy w- super open set is fuzzy rw- super open .But converse may not be true. For the fuzzy set  $B$  defined by  $B(a)=0.5$  and  $B(b)=0.7$  in the fuzzy topological  $(X, \mathfrak{T})$  of example 6.1.1 is fuzzy rg- super open but not fuzzy regular super open.

**Theorem 3.5:** A fuzzy set  $A$  of a fuzzy rw-super open if and only if  $F \leq \text{int}(A)$  whenever  $F \leq A$  and  $F$  is fuzzy regular semi super open.

Proof: Obvious.

**Theorem 3.6:** Let  $A$  be a fuzzy rw -super open set in a fuzzy topological spaces  $(X, \mathfrak{T})$  and  $\text{int}(A) \leq B \leq A$  then  $B$  is fuzzy rw- super open.

**Proof :** Obvious .

**Theorem 3.7:** Let  $(X, \mathfrak{T})$  be a fuzzy topological space and  $\text{FC}(X)$  be the family of all fuzzy super closed sets of  $X$  .Then  $\text{FRSSO}(X) \subseteq \text{FSC}(X)$  if and only if every fuzzy subset of  $X$  is fuzzy rw- super closed.

**Proof:** Necessity: Suppose that  $\text{FRSO}(X) \subseteq \text{FC}(X)$  and that  $A \leq U \in \text{FRSSO}(X)$  then  $\text{cl}(A) \leq \text{cl}(U) = U$  and  $A$  is fuzzy rw -super closed.

Sufficiency: Suppose that every fuzzy subset of  $X$  is fuzzy rw- super closed .If  $U \in \text{FRSSO}(X)$  then since  $U \leq U$  and  $U$  is fuzzy rw -super closed,  $\text{cl}(U) \leq U$  and  $U \in \text{FC}(X)$ . Thus  $\text{FRSO}(X) \subseteq \text{FC}(X)$ .

**Theorem 3.8:** Let  $A$  be a fuzzy w- super closed set in a fuzzy topological space  $(X, \mathfrak{T})$  and  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is a fuzzy almost super irresolute and fuzzy super closed mappings then  $f(A)$  is fuzzy rw - super closed in  $Y$ .

**Proof:** If  $f(A) \leq G$  where  $G \in \text{FRSSO}(Y)$ . Then  $A \leq f^{-1}(G) \in \text{FSSO}(X)$  and hence  $\text{cl}(A) \leq f^{-1}(G)$ , because  $A$  is a fuzzy w- super closed in  $X$ . Since  $f$  is fuzzy super closed,  $f(\text{cl}(A))$  is a fuzzy super closed set in  $Y$ . It follows that  $\text{cl}(f(A)) \leq \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \leq G$ . Thus  $\text{cl}(f(A)) \leq G$  and  $f(A)$  is a fuzzy rw –super closed set in  $Y$ .

**Definition 3.2:** A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is said to be fuzzy regular semi super irresolute if the inverse image of each fuzzy regular semi super open in  $X$ .

**Theorem 3.9:** Let  $A$  be the fuzzy rw -super closed set in a fuzzy topological space  $(X, \mathfrak{T})$  and  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is a fuzzy regular semi super irresolute and fuzzy super closed mapping then  $f(A)$  is fuzzy rw super closed sets in  $Y$ .

**Proof:** If  $f(A) \leq G$  where  $G \in \text{FRSO}(Y)$  then  $A \leq f^{-1}(G) \in \text{FRSO}(X)$  and hence  $\text{cl}(A) \leq f^{-1}(G)$  because  $A$  is fuzzy rw -super-closed in  $X$ . Since  $f$  is fuzzy closed  $f(\text{cl}(A))$  is a fuzzy closed set in  $Y$ . It follows that  $\text{cl}(f(A)) \leq \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \leq G$  thus  $\text{cl}(f(A)) \leq G$  and  $f(A)$  is fuzzy rw -super closed sets in  $Y$ .

**Definition 3.3:** A collection  $\{G_\alpha: \alpha \in \Lambda\}$  of fuzzy rw- super open sets in a fuzzy topological space  $(X, \mathfrak{T})$  is called a fuzzy rw- super open cover of a fuzzy set  $A$  of  $X$  if  $A \leq \bigcup \{G_\alpha: \alpha \in \Lambda\}$ .

**Definition 3.4:** A fuzzy set of a topological space  $(X, \mathfrak{T})$  is said to be fuzzy rw -super compact if every fuzzy rw –super open cover of  $X$  has a finite sub cover.

**Definition 3.5:** A fuzzy topological space  $(X, \mathfrak{T})$  is said to be fuzzy rw- super compact relative to  $X$ , if for every collection  $\{G_\alpha: \alpha \in \Lambda\}$  of fuzzy rw -super open subsets of  $X$  such that  $A \leq \bigcup \{G_{\alpha_j}: \alpha_j \in \Lambda_0\}$ .

**Definition 3.6:** A crisp subset of  $A$  of a fuzzy topological space  $(X, \mathfrak{T})$  is said to be fuzzy rw –super compact if  $A$  is fuzzy rw- super compact as a fuzzy subspace of  $X$ .

**Theorem 3.10:** Fuzzy rw-super closed crisp subsets of a fuzzy rw-super compact space are fuzzy rw-super compact relative to X.

**Proof:** Let A be a fuzzy rw super-closed crisp set off a fuzzy rw- super compact space  $(X, \mathfrak{T})$ . Then  $1 - A$  is fuzzy rw –super open in X .Let  $G = \{G_\alpha: \alpha \in \Lambda\}$ . Be a cover of A fuzzy rw- super open sets in X .Then the family  $\{G, 1-A\}$  is a fuzzy rg- super open in X is fuzzy rw –super compact.it had sub cover  $\{G_{\alpha_1} G_{\alpha_2} G_{\alpha_3} \dots G_{\alpha_n}\}$ . If the sub cover contain  $1-A$ , we discard it thus we have obtained a finite fuzzy rg-open sub cover of A is fuzzy rw -super-compact relative to X.

#### IV. FUZZY RW-SUPER CONTINUOUS MAPPINGS

In the present section we introduce the concept of fuzzy regular super w-Super continuous mappings in fuzzy topology and obtained some of its basic properties.

**Definition 4.1:** A fuzzy set A of topological spaces  $(X, \mathfrak{T})$  is called fuzzy regular w- super continuous mapping (written as fuzzy rw - super continuous mapping) if the inverse image of every fuzzy closed set of Y is fuzzy rw –super closed in X.

**Theorem 4.2:**A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is fuzzy rw-super continuous if and only if the inverse image of every fuzzy super open set of Y is fuzzy rw- super open in X.

**Proof:** It is obvious because  $f^{-1}(1-A) = 1-f^{-1}(A)$  for every fuzzy set A of Y.

**Remark 4.2:** Every fuzzy w-super continuous mapping is rw -super continuous but its converse may not be true for,

**Example 4.1:** Let  $X = \{a, b\}$  and  $Y = \{x, y\}$  the fuzzy sets U and V be defined as follows;

$$U(a)=0.7, \quad U(b)=0.6$$

$$V(x)=0.2, \quad V(y)=0.2$$

Let  $\mathfrak{T} = \{0, U, 1\}$  and  $\sigma = \{0, V, 1\}$  be a fuzzy topology on X and Y respectively .Then the mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is fuzzy rw- super continuous mapping but not fuzzy w-super continuous mapping.

**Remark 4.2:** Every fuzzy rw -super continuous mapping is rg-super continuous but its converse may not be true for,

**Example 4.2:** Let  $X = \{a, b\}$  and  $Y = \{x, y\}$  the fuzzy sets U and V be defined as follows;

$$U(a)=0.7, \quad U(b)=0.6$$

$$V(x)=0.2, \quad V(y)=0.2$$

Let  $\mathfrak{T} = \{0, U, 1\}$  and  $\sigma = \{0, V, 1\}$  be a fuzzy topology on X and Y respectively .Then the mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is fuzzy rw- super continuous mapping but not fuzzy w-super continuous mapping.

**Theorem 4.2:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is fuzzy w-super continuous then for each fuzzy point  $x_\beta$  of X and each fuzzy open set B,  $f(x_\beta) \in B$  then there exists a fuzzy rw –super open set A such that  $x_\beta \in A$  and  $f(A) \leq B$ .

**Proof:** Let  $x_\beta$  be fuzzy point of X and B be a fuzzy open set such that  $f(x_\beta) \in B$  put  $B = f^{-1}(A)$ , then by the hypothesis A is a fuzzy rw- super open set of X such that  $x_\beta \in A$  and  $f(A) = f(f^{-1}(B)) \leq B$ .

**Theorem 4.3:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is fuzzy rw super continuous then for each fuzzy point  $x_\beta$  of X and each fuzzy open set B of Y such that,  $f(x_\beta) \in B$  then there exists a fuzzy rw –super open set A such that  $x_\beta \in A$  and  $f(A) \leq B$ .

**Proof:** Let  $x_\beta$  be fuzzy point of X and B be a fuzzy super open set such that  $f(x_\beta) \in B$  put  $B = f^{-1}(A)$ , then by the hypothesis A is a fuzzy rw- super open set of X such that  $x_\beta \in A$  and  $f(A) = f(f^{-1}(B)) \leq B$ .

**Definition 4.2:** Let  $(X, \mathfrak{T})$  be a fuzzy topological .The rw –super closure of the fuzzy set A of X denoted by rw –super  $cl(A)$  is defined as follows  $rw\text{-super } cl(A) = \inf\{B: B \geq A, B \text{ is fuzzy rw- super closed set of } X\}$ .

**Remark 4.3:** It is clear that  $A \leq rw\text{-super } cl(A) \leq cl(A)$  for any fuzzy set  $A$  of  $X$ .

**Theorem 4.4:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is fuzzy  $rw\text{-super}$  continuous then  $f(rw\text{-super } cl(A)) \leq cl(f(A))$  for every fuzzy set  $A$  of  $X$ .

**Proof:** Let  $A$  be a fuzzy set of  $X$ . Then  $cl(f(A))$  is a fuzzy super closed set of  $Y$ . Since  $f$  is fuzzy  $rw\text{-super}$  continuous  $f^{-1}(cl(f(A)))$  is fuzzy  $rw\text{-super}$  closed in  $X$ . Clearly  $A \leq f^{-1}(cl(f(A)))$  therefore,  $rw\text{-super } cl(A) \leq rw\text{-super } cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$ , hence  $f(rw\text{-super } cl(A)) \leq cl(f(A))$ .

**Remark 4.4:** The converse of the theorem 6.2.2 is not true .

**Definition 4.3:** A fuzzy topological space  $(X, \mathfrak{T})$  is said to be fuzzy  $rw\text{-super-}T_{1/2}$  if every fuzzy  $rw\text{-super}$  closed set in  $X$  is fuzzy super closed in  $X$ .

**Theorem 4.5:** Let  $f$  be a mapping from a fuzzy  $rw\text{-super-}T_{1/2}$  space  $(X, \mathfrak{T})$  to a fuzzy topological space  $(Y, \sigma)$  then the following condition are equivalent:

- (a)  $f$  is fuzzy super continuous.
- (b)  $f$  is fuzzy  $w\text{-super}$  continuous.
- (c)  $f$  is fuzzy  $rw\text{-super}$  continuous.

**Proof:** Obvious.

**Theorem 4.6:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is fuzzy  $rw\text{-super}$  continuous and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  is fuzzy super continuous .Then  $g \circ f: (X, \mathfrak{T}) \rightarrow (Z, \eta)$  is fuzzy  $rw\text{-super}$  continuous.

**Proof:** If  $A$  is fuzzy closed in  $Z$  then  $f^{-1}(A)$  is fuzzy closed in  $Y$  because  $g$  is fuzzy super continuous. Therefore  $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$  is fuzzy  $rw\text{-super}$  closed in  $X$ . Hence  $g \circ f$  is fuzzy  $rw\text{-super}$  continuous.

**Theorem 4.7:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  are two fuzzy  $rw\text{-super}$  continuous mapping and  $(Y, \sigma)$  is fuzzy  $rw\text{-super-}T_{1/2}$  super continuous. Then  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is fuzzy  $rw\text{-super}$  continuous.

**Proof:** Obvious.

**Theorem 4.8:** A fuzzy  $rw\text{-super}$  continuous image of a fuzzy  $rw\text{-super}$  compact space is fuzzy compact.

**Proof:** Let  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  be a fuzzy  $rw\text{-super}$  continuous mapping from a fuzzy  $rw\text{-super}$  compact space  $(X, \tau)$  on to a fuzzy topological space  $(Y, \sigma)$ . Let  $\{A_i; i \in \Lambda\}$  be a fuzzy super open cover of  $Y$ , then  $\{f^{-1}(A_i); i \in \Lambda\}$  is a fuzzy  $rw\text{-super}$  open cover of  $X$ . Since  $X$  is fuzzy  $rw\text{-super}$  compact it has a finite sub cover, say  $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3) \dots f^{-1}(A_n)\}$  since  $f$  is onto  $\{A_1, A_2, \dots, A_n\}$  is a fuzzy super open cover of  $Y$  so  $(Y, \sigma)$  is fuzzy compact.

**Definition 4.4:** A fuzzy topological space  $(X, \mathfrak{T})$  is fuzzy  $rw\text{-super}$  connected if there is no proper fuzzy set of  $X$  which is both fuzzy  $rw\text{-super}$  open and fuzzy  $rw\text{-super}$  closed.

**Remark 4.5:** Every fuzzy  $rw\text{-super}$  connected space is fuzzy super connected but he converse may not be true. For;

**Example 4.4:** Let  $X = \{a, b\}$  and  $A$  be defined as follows,  $A(a) = 0.5, A(b) = 0.7$ . Let  $\mathfrak{T} = \{0, A, 1\}$  be topology on  $X$ , then  $(X, \mathfrak{T})$  is fuzzy super connected but not fuzzy  $rw\text{-super}$  connected.

**Theorem 4.10:** If  $(X, \mathfrak{T})$  fuzzy  $rw\text{-super-}T_{1/2}$  connected if and only if it is fuzzy  $rw\text{-super}$  connected.

**Proof:** Obvious.

**Theorem 4.10:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  fuzzy  $rw\text{-super}$  continuous surjection and  $X$  is fuzzy  $rw\text{-super}$  connected then  $Y$  is fuzzy super connected.

**Proof:** Suppose  $Y$  is not fuzzy super connected .Then there exists a proper fuzzy set  $A$  of  $Y$  which is both fuzzy super open and fuzzy super closed, therefore  $f^{-1}(A)$  is proper fuzzy set of  $X$ , which is both fuzzy  $rw\text{-super}$  open and fuzzy  $rw\text{-super}$  closed, because  $f$  is fuzzy  $rw\text{-super}$  continuous surjection. Hence,  $X$  is not fuzzy  $rw\text{-super}$  connected, which is a contradiction

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