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Convexity Preserving C^2 Rational Quadratic Trigonometric Spline

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Abstract- A C^2 rational quadratic trigonometric spline interpolation has been studied using two kinds of rational quadratic trigonometric splines. It is shown that under some natural conditions the solution of the problem exists and is unique. The necessary and sufficient condition that constrain the interpolant curves to be convex in the interpolating interval or subinterval are derived. approximation properties has been discussed and confirms the expected approximation order is h^2.

Index Terms- Approximation, Constrained interpolation, Continuity, Convexity, Rational quadratic trigonometric spline, Shape parameter .

I. Introduction

During the recent years rational parametric spline have gained widespread acceptance for use in computer aided geometric designs.They have been studied by several authors (see [1], [4], [8], [10]), with special emphasis on the shape preserving properties. In [1] Duan have presented the construction and shape preserving analysis of a new weighted rational cubic interpolation and its approximation. Trigonometric splines serves as an alternative for polynomial splines for solving many problems of interpolation and have been studied from application point of view (see [2], [3], [5], [6], [7], [9]). Trigonometric splines behave in a better way for path approximation or scattered data interpolation. B-splines introduced by Schoenberg [9], have become immensely popular for applications in curve and surface generation problems.Keeping in a view of the above ideas and the applications of trigonometric splines, we have extended the ideas of Duan [1] and constructed convexity preserving rational trigonometric spline. We have introduced a weighted rational quadratic trigonometric spline interpolation and the C^2 continuity of rational trigonometric spline.The convexity control and approximation properties of C^2 rational quadratic trigonometric spline have been discribed.

II. A C^1 WEIGHTED RATIONAL QUADRATIC TRIGONOMETRIC INTERPOLATION

A weighted rational cubic spline interpolation based on function values and derivative was given in [1]. Given a data set $(t_i, f(t_i), d_i)$, $i = 0,1,...,n,n+1$,where $f(t_i)$ and d_i are the function values and the derivative values defined at knots, respectively, and $t_0 < t_1 < ... < t_n < t_{n+1}$ are the knots.

Let $h_i = t_{i+1} - t_i$, $\theta = \dfrac{(t - t_i)}{h_i}$, $t\varepsilon[t_i, t_{i+1}]$ and α_i, β_i are

$$P^*(t) = \frac{(1-\sin\frac{\pi\theta}{2})^2 \alpha_i f(t_i) + 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})U_i^* + 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})V_i^* + (1-\cos\frac{\pi\theta}{2})^2 \beta_i f(t_{i+1})}{\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2}} \quad (1)$$

where

$$U_i^* = (\alpha_i + \frac{\beta_i}{2}) f(t_i) + \frac{\alpha_i h_i d_i}{\pi} \qquad V_i^* = (\beta_i + \frac{\alpha_i}{2}) f(t_{i+1}) - \frac{\beta_i h_i d_{i+1}}{\pi}$$

This rational quadratic trigonometric spline $P^*(t)$ satisfies

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$$P^*(t_i) = f(t_i), \quad P^{*'}(t_i) = d_i, \ i = 0,1,...,n,n+1.$$

for the given data set $(t_i, f(t_i)), i = 0,1,...,n,n+1$

let

$$P_*(t) = \frac{(1-\sin\frac{\pi\theta}{2})^2 \alpha_i f(t_i) + 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})U_{i,*} + 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})V_{i,*} + (1-\cos\frac{\pi\theta}{2})^2 \beta_i f(t_{i+1})}{\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2}} \quad (2)$$

where

$$U_{i,*} = (\alpha_i + \frac{\beta_i}{2}) f(t_i) + \frac{\alpha_i h_i \Delta_i}{\pi} \qquad V_{i,*} = (\beta_i + \frac{\alpha_i}{2}) f(t_{i+1}) - \frac{\beta_i h_i \Delta_{i+1}}{\pi}$$

in which $\Delta_i = \frac{f(t_{i+1}) - f(t_i)}{h_i}$.

Obviously the spline $P_*(t)$ satisfies

$$P_*(t_i) = f(t_i), \qquad P_*'(t_i) = \Delta_i, \ i = 0,1,...,n.$$

It is called rational quadratic trigonometric spline based on function values.

The weighted rational quadratic trigonometric spline will be constructed by using the two kinds of rational trigonometric quadratic spline interpolant described above.let

$$P(t) = \lambda P^*(t) + (1-\lambda) P_*(t) \quad t \varepsilon [t_i, t_{i+1}], \quad i = 0,1,...,n-1. \quad (3)$$

where

$$P(t) = \frac{(1-sin\frac{\pi\theta}{2})^2 \alpha_i f(t_i) + 2sin\frac{\pi\theta}{2}(1-sin\frac{\pi\theta}{2})U_i + 2cos\frac{\pi\theta}{2}(1-cos\frac{\pi\theta}{2})V_i + (1-cos\frac{\pi\theta}{2})^2 \beta_i f(t_{i+1})}{\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2}}$$

(4)

and

$$U_i = (\alpha_i + \frac{\beta_i}{2}) f(t_i) + \frac{\alpha_i h_i}{\pi}(\lambda d_i + (1-\lambda)\Delta_i),$$
$$V_i = (\beta_i + \frac{\alpha_i}{2}) f(t_{i+1}) - \frac{\beta_i h_i}{\pi}(\lambda d_{i+1} + (1-\lambda)\Delta_{i+1}),$$

with the weight coefficient $\lambda \varepsilon R$.
This rational quadratic trigonometric spline $P(t)$ satisfies

$$P(t_i) = f(t_i), \ P'(t_i) = \lambda d_i + (1-\lambda)\Delta_i, \ i = 0,1,...,n.$$

III. A C^2 WEIGHTED RATIONAL QUADRATIC TRIGONOMETRIC SPLINE INTERPOLATION

We now follow the familiar procedure of allowing the derivative parameters $d_i, i=1,...,n-1.$ to be degrees of freedom which are constrained by the imposition of the C^2 continuity conditions

$$P^{''}(t_i^+) = P^{''}(t_i^-), \qquad i=1,...,n-1.$$

the conditions leads to the following continuous system of linear equations:

$$\begin{aligned}
&(\frac{\pi\alpha_{i-1}}{2\beta_{i-1}h_{i-1}})d_{i-1} + (\frac{\pi(\alpha_i+\beta_i)}{\alpha_i h_i} + \frac{\pi(\alpha_{i-1}+\beta_{i-1})}{\beta_{i-1}h_{i-1}})d_i + (\frac{\pi\beta_i}{2\alpha_i h_i})d_{i+1} = \\
&\frac{\pi^2}{2\lambda\alpha_i^{\ 2}h_i^{\ 2}}\{(\alpha_i+2\beta_i)\frac{h_i\alpha_i}{2}\Delta_i - 2(1-\lambda)(\alpha_i+\beta_i)(\frac{\alpha_i h_i}{\pi})\Delta_i - (1-\lambda)\frac{\alpha_i\beta_i}{\pi}\Delta_{i+1}\} \\
&+\frac{\pi^2}{2\lambda\beta_{i-1}^2 h_{i-1}^{\ \ 2}}\{(2\alpha_{i-1}+\beta_{i-1})\frac{\beta_{i-1}h_{i-1}}{2}\Delta_{i-1} - (1-\lambda)\beta_{i-1}\frac{\alpha_{i-1}h_{i-1}}{\pi}\Delta_{i-1} \\
&\qquad - 2(1-\lambda)(\alpha_{i-1}+\beta_{i-1})\frac{\beta_{i-1}h_{i-1}}{\pi}\Delta_i\}
\end{aligned} \tag{5}$$

i=1,.....,n-1.

Therefore, if the successive parameters $(\alpha_{i-1},\beta_{i-1})$ and (α_i,β_i) satisfy (5) at $i=1,2,...,n-1,$ namely, for the positive parameters α_{i-1},β_{i-1} and the selected β_i, if

$$\alpha_i = \frac{2\beta_i\{(\pi-2)\Delta_i - \Delta_{i+1} + 2\lambda(\Delta_i - d_i) + \lambda(\Delta_{i+1} - d_{i+1})\}}{h_i\alpha_{i-1}[(2-2\pi)\Delta_{i-1} + 4\Delta_i - 4\lambda(\Delta_i - d_i) - 2\lambda(\Delta_{i-1} - d_{i-1})] + \beta_{i-1}\{h_i[-\pi\Delta_{i-1} + 4\Delta_i - 4\lambda(\Delta_i - d_i)] + h_{i-1}[(4-\pi)\Delta_i - 4\lambda(\Delta_i - d_i)]\}}$$

then $P(t) \in C^2(t_0, t_n).$

IV. CONVEXITY CONTROL OF RATIONAL QUADRATIC TRIGONOMETRIC SPLINE

Positivity, monotonicity and convexity are basic and fundamental shapes, which normally arise in everyday scientific phenomena. To get condition for the interpolation to keep convex in the interpolating interval,consider the condition for the second order derivative to remain positive or negative in the interpolating interval, this task can be carried out simply by selecting suitable values of the parameter λ to satisfy the linear inequality.In this section we assume that the knots are equally spaced.

For simplicity of presentation let us assume a strictly convex set of data so that

$$\Delta_1 < \Delta_2 < ... < \Delta_n.$$

In a similar fashion, one can deal with a concave data so that

$$\Delta_1 > \Delta_2 > ... > \Delta_n.$$

For a convex interpolant P(t), it is then necessary that the derivative parameters should be such that

$$d_1 < \Delta_1 < ... < d_i < \Delta_i < ...\Delta_{n-1} < d_n < \Delta_n$$

and for concave data.

$$(d_1 > \Delta_1 > ... > d_i > \Delta_i > ...\Delta_{n-1} > d_n > \Delta_n)$$

Now $P(t)$ is convex if and only if

$$P^{''}(t) \geq 0$$

For $t\varepsilon[t_i, t_{i+1}]$, the second-order derivative $P^{''}(t)$ can be computed and has the form

$$P^{''}(t) = Z(\theta).(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})^{-3} \tag{6}$$

where $Z(\theta) = Q + R + T$

and

$$Q = (\frac{\pi}{2h_i})^2(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})^2\{(1-\sin\frac{\pi\theta}{2})^2(2U_i - \alpha_i f_i) + 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})(2V_i - \beta_i f_{i+1})$$
$$+2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})(2U_i - \alpha_i f_i - (2V_i - \beta_i f_{i+1})) + (1-\cos\frac{\pi\theta}{2})^2(-(2V_i - \beta_i f_{i+1}))$$
$$+2\sin\frac{\pi\theta}{2}(2U_i - \alpha_i f_i - (2V_i - \beta_i f_{i+1})) + 2\cos\frac{\pi\theta}{2}(-(2U_i - \alpha_i f_i)) + 2(V_i - U_i + \alpha_i f_i)\}$$

$$R = (\frac{\pi}{2h_i})^2(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})(\alpha_i \sin\frac{\pi\theta}{2} - \beta_i \cos\frac{\pi\theta}{2})\{2\cos\frac{\pi\theta}{2}(U_i - \alpha_i f_i)$$
$$+2\sin\frac{\pi\theta}{2}\cos\frac{\pi\theta}{2}(2V_i - \beta_i f_{i+1} - (2U_i - \alpha_i f_i)) + 2\sin\frac{\pi\theta}{2}(\beta_i f_{i+1} - V_i)\}$$

$$T = (\frac{\pi}{2h_i})^2(\alpha_i \sin\frac{\pi\theta}{2} - \beta_i \cos\frac{\pi\theta}{2})^2\{(1-\sin\frac{\pi\theta}{2})^2\alpha_i f_i + 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})U_i$$
$$+2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})V_i + (1-\cos\frac{\pi\theta}{2})^2\beta_i f_{i+1}\}$$

The sufficient and necessary condition for the interpolating function $P(t)$ defined by (4) to be convex on $[t_i, t_{i+1}]$ is the positive parameter λ satisfy

$$\frac{2\beta_i h_i \Delta_{i+1} - \pi\alpha_i f_{i+1}}{2\beta_i h_i (\Delta_{i+1} - d_{i+1})} \le \lambda \le \frac{2\alpha_i h_i \Delta_i + \pi\beta_i f_i}{2\alpha_i h_i (\Delta_i - d_i)} \tag{7}$$

Here it is observed that if the data are positive/convex then the interpolant will be positive/convex. Thus we have proved the following theorem

Theorem 1.Given $(t_i, f_i, d_i), i = 0,1,...,n,n+1$,the necessary and sufficient condition for the interpolation defined by (3) to be convex on $[t_i, t_{i+1}]$ is that the given data and the positive parameter λ satisfy

$$\frac{2\beta_i h_i \Delta_{i+1} - \pi\alpha_i f_{i+1}}{2\beta_i h_i (\Delta_{i+1} - d_{i+1})} \le \lambda \le \frac{2\alpha_i h_i \Delta_i + \pi\beta_i f_i}{2\alpha_i h_i (\Delta_i - d_i)} \tag{8}$$

V. NUMERICAL EXAMPLE

Example:Let $f(t) = \cos^2(\pi t/6)$, $t\varepsilon[1.5, 4.5]$ with interpolating knots at $t_0 = 1.5$, $t_1 = 2.25$, $t_2 = 3.00$, $t_3 = 3.75$, $t_4 = 4.5$, $h = 0.75$. let $\lambda = 0.99$, and let $f(t)$ is the function being interpolated.Denote the corresponding C^2-continuous interpolating function defined by (4) in $[1.5, 4.5]$ by $p(t)$ since the interpolating data and parameter λ satisfy the condition of theorem 1. as shown in figure 1.

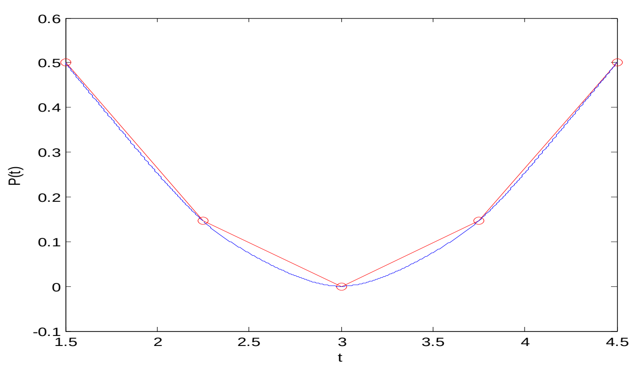

Figure 1: graph of $P(t)$

VI. APPROXIMATION PROPERTIES OF THE WEIGHTED RATIONAL QUADRATIC TRIGONOMETRIC INTERPOLATION

To estimate error of the weighted rational quadratic trigonometric interpolating function defined by (4), since the interpolation is local,without loss of generality, we consider the error in the subinterval $[t_i, t_{i+1}]$. When $f(t)\varepsilon C^2[t_0, t_n]$ and P(t) is the rational quadratic trigonometric spline interpolating function of $f(t)$ in $[t_i, t_{i+1}]$. It is easy to see that this type of interpolation is exact for $f(t)$, the polynomial being interpolated,in which the degree is no more than 1. Consider the case when the knots are equally spaced,namely, $h_i = h = \frac{t_n - t_0}{n}$ for all $i = 1,2,...,n,$ using the Peano-Kernel Theorem in Schultz [13)] gives the following

$$R[f] = f(t) - P(t) = \int_{t_i}^{t_{i+1}} f^{(2)}(\tau) R_t[(t-\tau)_+] \mathrm{d}\tau, \quad t\varepsilon[t_i, t_{i+1}] \tag{9}$$

where

$$R_t[(t-\tau)_+] = \begin{cases} p(\tau) & t_i < \tau < t \\ q(\tau) & t < \tau < t_{i+1} \\ r(\tau) & t_{i+1} < \tau < t_{i+2} \end{cases}$$

where

$$p(\tau) = (t-\tau) - \{[\frac{(1-\lambda)\alpha_i}{\pi} 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2}) + (\beta_i + \frac{\alpha_i}{2}) 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}) + (1-\cos\frac{\pi\theta}{2})^2](t_{i+1}-\tau) - \frac{\beta_i h_i}{\pi} 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})\} / (\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2}),$$
$$t_i < \tau < t; \quad (10)$$

$$q(\tau) = -\{[\frac{(1-\lambda)\alpha_i}{\pi} 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2}) + (\beta_i + \frac{\alpha_i}{2}) 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}) + (1-\cos\frac{\pi\theta}{2})^2 \beta_i](t_{i+1}-\tau) - \frac{\beta_i h_i}{\pi} 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})\} / (\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2}),$$
$$t < \tau < t_{i+1}; \quad (11)$$

$$r(\tau) = \frac{\frac{\beta_i(1-\lambda)}{\pi} 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})(t_{i+2}-\tau)}{(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})}, \quad t_{i+1} < \tau < t_{i+2}; \quad (12)$$

Then

$$\| R[f] \| = \| f(t) - P(t) \| \le \| f^2(t) \| \{ \int_{t_i}^{t} | p(\tau) | \, d\tau + \int_{t}^{t_{i+1}} | q(\tau) | \, d\tau + \int_{t_{i+1}}^{t_{i+2}} | r(\tau) | \, d\tau \} \quad (13)$$

for $\lambda \le 1$ $r(\tau) \ge 0$ for all $\tau \varepsilon [t_{i+1}, t_{i+2}]$, thus

$$\int_{t_{i+1}}^{t_{i+2}} | r(\tau) | \, d\tau = h^2 \frac{\frac{\beta_i(1-\lambda)}{\pi} \cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})} \quad (14)$$

for $q(\tau)$, since

$$q(t)=-h\{(1-\theta)[\frac{\alpha_i(1-\lambda)}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})+(\beta_i+\frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})+\beta_i(1-\cos\frac{\pi\theta}{2})^2]$$
$$-\frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})\}/(\alpha_i\cos\frac{\pi\theta}{2}+\beta_i\sin\frac{\pi\theta}{2})\le 0 \quad (15)$$

and

$$q(t_{i+1})=h\frac{\frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{(\alpha_i\cos\frac{\pi\theta}{2}+\beta_i\sin\frac{\pi\theta}{2})}\ge 0 \quad (16)$$

It is easy to see that the root τ^* of $q(\tau)$ is

$$\tau^*=t_{i+1}-h\frac{\frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{[\frac{\alpha_i(1-\lambda)}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})+(\beta_i+\frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})+\beta_i(1-\cos\frac{\pi\theta}{2})^2]}$$

Thus

$$\int_t^{t_{i+1}}|q(\tau)|\,d\tau=\int_t^{\tau^*}-q(\tau)d\tau+\int_{\tau^*}^{t_{i+1}}q(\tau)d\tau=h^2[Q_1]$$

$$Q_1=h^2\{[\frac{\alpha_i(1-\lambda)}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})+(\beta_i+\frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})+\beta_i(1-\cos\frac{\pi\theta}{2})^2](\frac{(1-\theta)^2}{2}+z^2)$$
$$+(\frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}))(2z-(1-\theta))\}/(\alpha_i\cos\frac{\pi\theta}{2}+\beta_i\sin\frac{\pi\theta}{2}) \quad (17)$$

$$z=\frac{\beta_i 2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{\frac{\alpha_i(1-\lambda)}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2})+(\beta_i+\frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})+\beta_i(1-\cos\frac{\pi\theta}{2})^2} \quad (18)$$

similarly $\because \quad p(t)=q(t)\le 0$

$$p(t_i) = h[\theta - \{\frac{\alpha_i(1-\lambda)}{\pi} 2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2}) + (\beta_i + \frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}) + \beta_i(1-\cos\frac{\pi\theta}{2})^2 - \frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})\}/(\alpha_i\cos\frac{\pi\theta}{2} + \beta_i\sin\frac{\pi\theta}{2})] \geq 0 \quad (19)$$

and the root τ_* of $p(\tau)\, in\, [t_i, t]$ is

$$\tau_* = t_{i+1} - h\frac{M}{N}$$

where

$$M = h[(\theta-1)(\alpha_i\cos\frac{\pi\theta}{2} + \beta_i\sin\frac{\pi\theta}{2}) + \frac{\beta_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})] \quad (20)$$

$$N = [\frac{\alpha_i(1-\lambda)}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2}) + (\beta_i + \frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}) + \beta_i(1-\cos\frac{\pi\theta}{2})^2 - (\alpha_i\cos\frac{\pi\theta}{2} + \beta_i\sin\frac{\pi\theta}{2})] \quad (21)$$

so that

$$\int_{t_i}^{t} |\, p(\tau)\,|\, \mathrm{d}\tau = \int_{t_i}^{\tau^*} p(\tau)\mathrm{d}\tau + \int_{\tau^*}^{t} -p(\tau)\mathrm{d}\tau$$

$$= h^2\{z_1 + z_2\frac{[\frac{(1-\lambda)\alpha_i}{\pi}2\sin\frac{\pi\theta}{2}(1-\sin\frac{\pi\theta}{2}) + (\beta_i + \frac{\alpha_i}{2})2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2}) + (1-\cos\frac{\pi\theta}{2})^2](t_{i+1}-\tau) - \frac{\beta_i h_i}{\pi}2\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{(\alpha_i\cos\frac{\pi\theta}{2} + \beta_i\sin\frac{\pi\theta}{2})}$$

$$+ z_3\frac{\frac{2\beta_i}{\pi}\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{(\alpha_i\cos\frac{\pi\theta}{2} + \beta_i\sin\frac{\pi\theta}{2})}\}$$

$$= h^2 Q_2$$

where

$$z_1 = [\frac{\theta^2}{2} - ((1-\theta) - \frac{M}{N})^2]$$

$$z_2 = [\frac{M}{N}^2 - \frac{1}{2} - \frac{(1-\theta)^2}{2}]$$

$$z_3 = [(2(1-\frac{M}{N}) - \theta)]$$

$$\| R[f] \| = \| f(t) - P(t) \| \leq h^2 \| f^2(t) \| w(\theta, \alpha_i, \beta_i)$$

where

$$w(\theta, \alpha_i, \beta_i) = [\frac{\frac{\beta_i(1-\lambda)}{\pi}\cos\frac{\pi\theta}{2}(1-\cos\frac{\pi\theta}{2})}{(\alpha_i \cos\frac{\pi\theta}{2} + \beta_i \sin\frac{\pi\theta}{2})} + Q_1 + Q_2] \quad (22)$$

where $w(\theta, \alpha_i, \beta_i)$ is a constant depending upon $\theta, \alpha_i, \beta_i$.

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