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Batch arrival Feedback Queue with Additional Multi Optional Service and Multiple Vacation

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Abstract- Batch arrival feedback queue with additional multi optional service and multiple vacation is considered. All the arriving customers demand first essential service and only some of them demand second optional service. After the completion of second service, customer may feedback to the tail of original queue to repeat the service until it is successful or may depart forever from the system. If there is no customer in the queue the server goes on multiple vacation. Service times are gernerally distributed and vacation time is exponentially distributed. The time dependent probability generating functions has been obtained in terms of their Laplace transforms and the corresponding steady state results are obtained explicitly. Mean queue length and mean waiting time are computed.

***Index Terms*-** Batch arrival feedback, multi optional service, multiple vacation.

I. Introduction

In many examples such as production system, bank services, computer and communication network, besides feedback the system have vacation. Levy and Yechiali [7], Borthakur and Choudhury [2], Madan and Jehad [10], Madan and Anabosi [9], Badamchi Zadeh and Shankar [1] and many others have studied vacation queues.

Igaki [5], Chae.et al [3] have studied queues with generalized vacation. Tian and Zhang [14] analyzed the discrete-time G/Geo/1 queue with multiple vacation

Medhi [11], Choudhury [4] , Kalyanaraman and Pazhani Bala Murugan [6], Roobala and Udayachandrika[12] have studied single server batch arrival queuing system with an additional service channel. Madan[8] had discussed an M/G/1 queue with second muti optional service.

Thangaraj and Vanitha [13], have studied a two phase M/G/1 feedback queue with multiple vacation. In this paper we analyse a single server queue with batch arrival Poisson input with two phases of heterogeneous service. The first phase is essential and the second phase has multi optional service.

II. Mathematical description of the model

- Customers arrive in batches according to a compound Poisson process with rate λ. Let X_k denote the number of customers belonging to the k^{th} arrival batch, where X_k, k = 1, 2, 3, … are with a common distribution

$Pr[X_k = n] = a_n$, n = 1, 2, 3, … and $X(z) = \sum_{n=1}^{\infty} a_n z^n$ denotes the probability generating function of X.

- The server provides the first essential service to all arriving customers. Its service time has general distribution with distribution function $B_0(x)$, density function $b_0(x)$, mean service times $\frac{1}{\mu_0}$ and hazard rate function $\mu_0(x)$.
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- As soon as the first service of a customer is completed, then with probability r_k $(1 \le k \le m)$ the customer may opt for a certain second optional service from m kinds of different service, or else, with probability $r_0 = 1 - \sum_{k=1}^{m} r_k$, he may opt to leave the system. The second service time has general distribution. with distribution function $B_k(x)$, density function, $b_k(x)$, mean service times $\frac{1}{\mu_k}$ and the hazard rate function $\mu_k(x)$, where $1 \le k \le m$.
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- After completion of second service, if the customer is dissatisfied with its service, then with probability p he may join the tail of the original queue as a feedback customer for receiving another regular service. Otherwise the customer may depart forever from the system with probability q = 1 − p. The customers are served according to First In First Out rule.
- If there is no customer waiting in the queue, then the server goes for a vacation. The vacation periods are exponentially distributed with mean vacation time $\frac{1}{\gamma}$. On returning from vacation if the server again founds no customer in the queue, then it goes for another vacation. So the server takes multiple vacations.

III. Equations Governing the System

- Let $P_n^{(0)}(x,t)$ be the probability that at time t, there are n customers in the queue excluding the one being provided the first essential service with elapsed service time x.
- $P_n^{(k)}(x,t)$ be the probability that there are n customers in queue excluding the customer being provided the k^{th} optional service, with elapsed service time x.
- $V_n(t)$ be the probability that at time t, there are n customers in the queue and the server is on vacation.

Assume that initially there is no customer in the system and the server is under vacation. Then the initial conditions are

$V_0(0) = 1$, $V_n(0) = 0$ and $P_n^{(j)}(0) = 0$ for $n \geq 0$ and $j = 1, 2, \ldots, k$ (1)

IV. Generating Functions of Queue Length

Define the probability generating functions

$$P_q^{(0)}(x,z,t) = \sum_{n=0}^{\infty} P_n^{(0)}(x,t)\, z^n, \qquad P_q^{(0)}(z,t) = \sum_{n=0}^{\infty} P_n^{(0)}(t)\, z^n$$

$$P_q^{(k)}(x,z,t) = \sum_{n=0}^{\infty} P_n^{(k)}(x,t)\, z^n, \qquad P_q^{(k)}(z,t) = \sum_{n=0}^{\infty} P_n^{(k)}(t)\, z^n$$

and $$V(z,t) = \sum_{n=0}^{\infty} V_n(t)\, z^n \tag{2}$$

which are convergent inside the circle given by | z | = 1.

Laplace transform of the differential difference equations that govern the model under consideration are

$$\frac{\partial}{\partial x}\bar{P}_0^{(0)}(x,s) + (s+\lambda+\mu_0(x))\bar{P}_0^{(0)}(x,s) = 0 \tag{3}$$

$$\frac{\partial}{\partial x}\bar{P}_n^{(0)}(x,s) + (s+\lambda+\mu_0(x))\bar{P}_n^{(0)}(x,s) = \lambda\sum_{i=1}^{n} a_i\,\bar{P}_{n-i}^{(0)}(x,s),\ n \geq 1 \tag{4}$$

$$\frac{\partial}{\partial x}\bar{P}_0^{(k)}(x,s) + (s+\lambda+\mu_k(x))\bar{P}_0^{(k)}(x,s) = 0,\quad k = 1, 2, \ldots, m \tag{5}$$

$$\frac{\partial}{\partial x}\bar{P}_n^{(k)}(x,s) + (s+\lambda+\mu_k(x))\bar{P}_n^{(k)}(x,s) = \lambda\sum_{i=1}^{n} a_i\,\bar{P}_{n-i}^{(k)}(x,s),\quad n \geq 1,\ k = 1, 2, \ldots m \tag{6}$$

$$(s+\lambda+\gamma)\bar{V}_0(s) = 1 + r_0\int_0^{\infty}\bar{P}_0^{(0)}(x,s)\mu_0(x)\,dx + q\sum_{k=1}^{m}\int_0^{\infty}\bar{P}_0^{(k)}(x,s)\mu_k(x)\,dx + \gamma\,\bar{V}_0(s) \tag{7}$$

$$(s+\lambda+\gamma)\bar{V}_n(s) = \lambda\sum_{i=1}^{n} a_i\,\bar{V}_{n-i}(s),\quad n \geq 1 \tag{8}$$

$$\bar{P}_n^{(0)}(0,s) = q\sum_{k=1}^{m}\int_0^{\infty}\bar{P}_{n+1}^{(k)}(x,s)\mu_k(x)\,dx + r_0\int_0^{\infty}\bar{P}_{n+1}^{(0)}(x,s)\mu_0(x)\,dx$$

$$+ (1-q) \sum_{k=1}^{m} \int_0^{\infty} \bar{P}_n^{(k)}(x,s)\, \mu_k(x)\, dx + \gamma\, \bar{V}_{n+1}(s), \quad n \geq 0 \qquad (9)$$

$$\bar{P}_n^{(k)}(0,s) = r_k \int_0^{\infty} \bar{P}_n^{(0)}(x,s)\, \mu_0(x)\, dx, \qquad n \geq 0, \quad k = 1, 2, \ldots, m \qquad (10)$$

Multiplying equation (4) by z^n, summing over n from 1 to ∞ and adding the result to equation (3) we get

$$\frac{\partial}{\partial x} \bar{P}_q^{(0)}(x,z,s) + (s + \lambda - \lambda(X(z)) + \mu_0(x))\, \bar{P}_q^{(0)}(x,z,s) = 0 \qquad (11)$$

By similar operations equations (5) and (6) yield

$$\frac{\partial}{\partial x} \bar{P}_q^{(k)}(x,z,s) + (s + \lambda - \lambda(X(z)) + \mu_k(x))\, \bar{P}_q^{(k)}(x,z,s) = 0 \qquad (12)$$

and equations (7) and (8) yield

$$(s + \lambda - \lambda(X(z)) + \gamma)\, \bar{V}(z,s) = 1 + r_0 \int_0^{\infty} \bar{P}_0^{(0)}(x,s)\, \mu_0(x)\, dx + q \sum_{k=1}^{m} \int_0^{\infty} \bar{P}_0^{(k)}(x,s)\, \mu_k(x)\, dx + \gamma\, \bar{V}_0(s) \qquad (13)$$

From equations (9) and (10) we get

$$z\, \bar{P}_q^{(0)}(0,z,s) = [z + q(1-z)] \sum_{k=1}^{m} \int_0^{\infty} \bar{P}_q^{(k)}(x,z,s)\, \mu_k(x)\, dx - q \sum_{k=1}^{m} \int_0^{\infty} \bar{P}_0^{(k)}(x,s)\, \mu_k(x)\, dx + r_0 \left[\int_0^{\infty} \left(\bar{P}_0^{(0)}(x,z,s) - P_0^{(0)}(x,s) \right) \mu_0(x)\, dx \right] + \gamma\, [\bar{V}(z,s) - \bar{V}_0(s)] \qquad (14)$$

and

$$\bar{P}_q^{(k)}(0,z,s) = r_k \int_0^{\infty} \bar{P}_q^{(0)}(x,z,s)\, \mu_0(x)\, dx, \qquad n \geq 0,\ k = 1, 2, \ldots, m \qquad (15)$$

Solution of equation (11) is

$$\bar{P}_q^{(0)}(x,z,s) = \bar{P}_q^{(0)}(0,z,s)\, e^{-(s+\lambda-\lambda(X(z)))x - \int_0^x \mu_0(x)\, dx} \qquad (16)$$

Integrating equation (16) with respect to x from 0 to ∞ we obtain

$$\bar{P}_q^{(0)}(z,s) = \bar{P}_q^{(0)}(0,z,s) \left[\frac{1 - \bar{B}_0(s + \lambda - \lambda(X(z)))}{s + \lambda - \lambda(X(z))} \right] \qquad (17)$$

$$\int_0^\infty \bar{P}_q^{(0)}(x,z,s)\,\mu_0\,dx = \bar{P}_q^{(0)}(0,z,s)\int_0^\infty e^{-(s+\lambda-\lambda(x(z)))x}\,d\,B_0(x)$$

$$= \bar{P}_q^{(0)}(0,z,s)\,\bar{B}_0(s+\lambda-\lambda(X(z))) \quad (18)$$

Similarly from equation (12) we obtain

$$\bar{P}_q^{(k)}(x,z,s) = \bar{P}_q^{(k)}(0,z,s)\,e^{-(s+\lambda-\lambda(x(z)))x-\int_0^x \mu_k(x)dx} \quad k = 1, 2, \ldots, m \quad (19)$$

$$\bar{P}_q^{(k)}(z,s) = \bar{P}_q^{(k)}(0,z,s)\left[\frac{1-\bar{B}_k(s+\lambda-\lambda(X(z)))}{s+\lambda-\lambda(X(z))}\right] \quad (20)$$

$$\int_0^\infty \bar{P}_q^{(k)}(x,z,s)\,\mu_k(x)\,dx = \bar{P}_q^{(k)}(0,z,s)\,\bar{B}_k(s+\lambda-\lambda(X(z))),\ k = 1, 2, \ldots, m \quad (21)$$

Using equations (13),(15),(18) and (21), in equation (14) and simplifying we get

$$\bar{P}_q^{(0)}(0,z,s) = \left[\frac{1-(s+\lambda-\lambda(X(z)))\,\bar{V}(z,s)}{D(z,s)}\right] \quad (22)$$

where $D(z,s) = z - [z + q(1-z)]\sum_{k=1}^{m} r_k\,\bar{B}_0(s+\lambda-\lambda(X(z)))\,\bar{B}_k(s+\lambda-\lambda(X(z)))$

$$- r_0\,\bar{B}_0(s+\lambda-\lambda(X(z))) \quad (23)$$

Expressions of $\bar{P}_q^{(0)}(z,s)$, $\bar{P}_q^{(k)}(0,z,s)$, $\bar{P}_q^{(k)}(z,s)$ are obtained as

$$\bar{P}_q^{(0)}(z,s) = \left[\frac{1-(s+\lambda-\lambda(X(z)))\,\bar{V}(z,s)}{D(z,s)}\right]\left[\frac{1-\bar{B}_0(s+\lambda-\lambda(X(z)))}{s+\lambda-\lambda(X(z))}\right] \quad (24)$$

$$\bar{P}_q^{(k)}(0,z,s) = r_k\,\bar{B}_0(s+\lambda-\lambda(X(z)))\left[\frac{1-(s+\lambda-\lambda(X(z)))\,\bar{V}(z,s)}{D(z,s)}\right] \quad (25)$$

$$\bar{P}_q^{(k)}(z,s) = r_k\,\bar{B}_0(s+\lambda-\lambda(X(z)))\left[\frac{1-\bar{B}_k(s+\lambda-\lambda(X(z)))}{s+\lambda-\lambda(X(z))}\right]\left[\frac{1-(s+\lambda-\lambda(X(z)))\,\bar{V}(z,s)}{D(z,s)}\right] \quad (26)$$

Let $\bar{P}_q(z,s)$ denote the probability generating function of the number in the queue irrespective of the type of service being provided.Then

$$\bar{P}_q(z,s) = \bar{P}_q^{(0)}(z,s) + \bar{P}_q^{(k)}(z,s)$$

$$= \left[\frac{N(z,s)}{s+\lambda-\lambda(X(z))}\right]\left[\frac{1-(s+\lambda-\lambda(X(z)))\,\overline{V}(z,s)}{D(z,s)}\right] \quad (27)$$

where

$$N(z,s) = 1 - \overline{B}_0(s+\lambda-\lambda(X(z))) + r_k\,\overline{B}_0(s+\lambda-\lambda(X(z))) - r_k\,\overline{B}_0(s+\lambda-\lambda(X(z)))\,\overline{B}_k(s+\lambda-\lambda(X(z))) \quad (28)$$

It can be shown that the denominator of equation (27) has one zero inside the sunit circle | z | = 1, which is sufficient to determine the unknown $\overline{V}(z,s)$ appearing in the numerator. Therefore $\overline{P}_q(z,s)$ and $\overline{P}_q^{(0)}(z,s)$ and $\overline{P}_q^{(k)}(z,s)$ can be completely determined.

V. The Steady State Results

The steady state results can be obtained by applying the well-known Tauberian property.

$$\lim_{s\to 0} s\,\overline{f}(s) = \lim_{s\to\infty} f(t) \quad (29)$$

Thus, multiplying both sides of equation (27) by s and taking limit as $s \to 0$, applying property (29) and simplifying, we have

$$P_q(z) = V(z)\left[\frac{N(z)}{D(z)}\right] \quad (30)$$

where

$$N(z) = 1 - \overline{B}_0(\lambda-\lambda(X(z))) + r_k\,\overline{B}_0(\lambda-\lambda(X(z))),\ k = 1, 2, \ldots, m - r_k\,\overline{B}_0(\lambda-\lambda(X(z)))\,\overline{B}_k(\lambda-\lambda(X(z))) \quad (31)$$

$$D(z) = z - [z + q(1-z)]\sum_{k=1}^{m} r_k\,\overline{B}_0(\lambda-\lambda(X(z)))\,\overline{B}_k(\lambda-\lambda(X(z))) - r_0\,\overline{B}_0(\lambda-\lambda(X(z))) \quad (32)$$

$$P_q(1) = \frac{-\lambda E(X)\,[E(V_0) + \sum_{k=1}^{m} r_k E(V_k)]}{[1+\lambda E(X)E(V_0) - (1-q)\sum_{k=1}^{m} r_k + \lambda E(X)\sum_{k=1}^{m} r_k E(V_k)]}\,V(1) \quad (33)$$

By normalizing condition we must have $V(1) + P_q(1) = 1$.

Therefore adding V(1) to equation (33), equating to 1 and simplifying, we get

$$V(1) = \frac{1+\lambda E(X)E(V_0) - (1-q)\sum_{k=1}^{m} r_k + \lambda E(X)\sum_{k=1}^{m} r_k E(V_k)}{1-(1-q)\sum_{k=1}^{m} r_k} \quad (34)$$

where $\dfrac{1+\lambda E(X) E(V_0) - (1-q)\sum_{k=1}^{m} r_k + \lambda E(X) \sum_{k=1}^{m} r_k E(V_k)}{1-(1-q)\sum_{k=1}^{m} r_k} < 1$

emerges to be the stability condition under which the steady state solution exists.

We note that V(1) is the steady state probability that the server is under vacation. Consequently system utilization factor is given by

$$\rho = 1 - V(1)$$

$$= 1 - \left\{ \frac{1+\lambda E(X) E(V_0) - (1-q)\sum_{k=1}^{m} r_k + \lambda E(X) \sum_{k=1}^{m} r_k E(V_k)}{1-(1-q)\sum_{k=1}^{m} r_k} \right\}$$

$$= \frac{-\lambda E(X)\left[E(V_0) + \sum_{k=1}^{m} r_k E(V_k)\right]}{1-(1-q)\sum_{k=1}^{m} r_k} \qquad (35)$$

VI. The Mean Number In the System

Let L_q denote the mean number of customers in the queue under steady state then

$$L_q = \frac{d}{dz} P_q(z) \text{ at } z = 1 \qquad (36)$$

since the formula gives 0/0 form, applying L'Hopital rule, we get

$$L_q = \lim_{z\to 1} \frac{d}{dz} P_q(z)$$

$$= \lim_{z\to 1} \frac{D'(z) N''(z) - N'(z) D''(z)}{2(D'(z))^2}$$

$$= \frac{D'(1) N''(1) - N'(1) D''(1)}{2(D'(1))^2} \qquad (37)$$

where

$$N'(1)\, V(1) = -\lambda E(X) \left[E(V_0) + \sum_{k=1}^{m} r_k E(V_k)\right] \left[\frac{1+\lambda E(V_0) - (1-q)\sum_{k=1}^{m} r_k + \lambda E(X) \sum_{k=1}^{m} r_k E(V_k)}{1-(1-q)\sum_{k=1}^{m} r_k}\right] \qquad (38)$$

$$N''(1)\, V(1) = -(\lambda E(X))^2 \left[E(V_0^2) + \sum_{k=1}^{m} 2r_k E(V_0) E(V_k) + r_k E(V_k^2)\right]$$

$$\left[\frac{1+\lambda E(X) E(V_0)-(1-q)\sum_{k=1}^{m} r_k+\lambda E(X)\sum_{k=1}^{m} r_k E(V_k)}{1-(1-q)\sum_{k=1}^{m} r_k}\right] \quad (39)$$

$$D'(1) = 1+\lambda E(X) E(V_0)-(1-q)\sum_{k=1}^{m} r_k+\lambda E(X)\sum_{k=1}^{m} r_k E(V_k) \quad (40)$$

$$D''(1) = (-\lambda E(X))^2 \sum_{k=1}^{m} [E(V_0)+2E(V_0)E(V_k)+E(V_k^2)] - \lambda E(X)\, 2p\sum_{k=1}^{m} r_k [E(V_0)+E(V_k)] \quad (41)$$

By substituting the above values of N′(1), N″(1), D′(1), D″(1) in equation (37) we obtain L_q.

If L denote the mean number in the system including the one in service, then we have

$$L = L_q + \rho \quad (42)$$

VII. THE MEAN WAITING TIME

The mean waiting time in the queue and in the system are respectively obtained by using

$$W_q = L_q / \lambda \text{ and} \quad (43)$$
$$W = L / \lambda \quad (44)$$

VIII. SPECIAL CASES

Case (i)

If p = 0, the system reduces to $M^X/G/1$ queue with multi-optional service and vacation.
Probability that the system busy is

$$P_q(1) = \frac{-\lambda E(X)\left[E(V_0)+\sum_{k=1}^{m} r_k E(V_k)\right]}{\left[1+\lambda E(X)\left(E(V_0)+\sum_{k=1}^{m} r_k E(V_k)\right)\right]} V(1) \quad (45)$$

Probability that the server is on vacation is

$$V(1) = 1+\lambda E(X)\left(E(V_0)+\sum_{k=1}^{m} r_k E(V_k)\right) \quad (46)$$

Expected queue size is obtained by substituting p = 0 in equation (36).

Case (ii)

Taking $r_1 = r_2, \ldots, r_m = 0$, in equations (33), (36), (42) and (44) we get the corresponding results for $M^X/G/1$ feedback queue with multiple vacation.

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