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Implementation of Spectrum Analyzer using GOERTZEL Algorithm

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Abstract- Spectrum analysis is very essential requirement in instrumentation and communication signal interception. Spectrum analysis is normally carried out by online or offline FFT processing. The Goertzel algorithm is a digital signal processing (DSP) technique for identifying frequency components of a signal. While the general Fast Fourier transform (FFT) algorithm computes evenly across the bandwidth of the incoming signal, the Goertzel algorithm looks at specific, predetermined frequency. The FPGA being capable of offering high frequency data paths become suitable for realizing high speed spectrum analysis algorithms. The objective of this thesis is implementing Goertzel algorithm as high Q band pass filter on FPGA reconfigurable architecture. A digital frequency synthesizer produces frequency sweep which will drive the digital mixer. The digital mixer output is given to the Goertzel algorithm block. This algorithm output will be given to peak detection logic. The peak detector block output will be used for spectrum computation. The top level module integrates all these modules with appropriate clock and control circuitry. The results will be demonstrated by applying the deterministic signals such as sine wave and also with random band limited signals. It will be aimed to achieve 32 steps in the band of operation for spectrum computation on Spartan 3E low cost FPGA.

Index Terms- Goertzel algorithm, Spectrum analysis, FPGA, Fast Fourier transform (FFT) algorithm

I. Introduction

The challenges with FFT based spectrum analyzers makes implementation of spectrum analysis difficult. Spectrum of a signal reveals the elements of the signal, and also the performance of the circuit producing them .Spectrum analyzer is a frequency-selective, peak-responding voltmeter which displays the amplitude of a sine wave. Spectrum analyzers are able to measure a large variety of input signals and in this way they are an invaluable tool for the RF design development and test laboratories, as well as having many applications for specialist field service.

Goertzel algorithm plays important role in the electronics industry for analyzing the frequency spectrum of radio frequency (RF) and audio signals and has some preferred properties such as high speed, low area and low power consumption. Goertzel algorithm is a recursive filter that aims at specified frequency in the spectrum. Spectrum analysis from the input signal has been replaced by the active development of a wide range of very specialized techniques and most of the existing spectrum analyzers are highly specific to a certain input signal and some research is pursued to integrate these techniques.

II. SPECTRUM ANALYSIS

The spectrum is the basic measurement of an FFT analyzer. It is simply the complex FFT. The spectral magnitude of the given frequency component is calculated by squaring and adding the sine and cosine coefficients for that frequency and taking the square root, giving the modulus of the complex FFT value. The phase spectrum, corresponding to the argument of the complex FFT value, can in principle be determined from the sine and cosine coefficients. (In most practical applications of the FFT, only the magnitude spectrum is of interest and phase information is not required). The magnitude is a real quantity and represents the total signal amplitude in each frequency bin, independent of phase. If there is phase information in the spectrum, i.e. the time record is triggered in phase with some component of the signal, then the real (cosine) or imaginary (sine) part or the phase may be displayed. The phase is simply the arctangent of the ratio of the imaginary and real parts of each frequency component. The phase is always relative to the start of the triggered time record.

Fourier's theorem states that any waveform in the time domain can be represented by the weighted sum of sine's and cosines. The FFT spectrum analyzer samples the input signal, computes the magnitude of its sine and cosine components, and displays the spectrum of these measured frequency components. For one thing, some measurements which are very hard in the time domain are very easy in the frequency domain. Consider the measurement of harmonic distortion. It's hard to quantify the distortion of a sine wave by looking at the signal on an oscilloscope. When the same signal is displayed on a spectrum analyzer, the harmonic frequencies and amplitudes are displayed with amazing clarity. Another example is noise analysis. Looking at an amplifier's output noise on an oscilloscope basically measures just the total noise amplitude. On a spectrum analyzer, the noise as a function of frequency is displayed. It may be that the amplifier has a problem only over certain frequency ranges. In the time domain it would be very hard to tell. Many of these measurements were once done using analog spectrum analyzers. In simple terms, an analog filter was used to isolate frequencies of interest. The signal power which passed through the filter was measured to determine the signal strength in certain frequency bands.

By tuning the filters and repeating the measurements, a spectrum could be obtained.

Types of Spectrum Analyzers:

a. *Scan Based SA*

b. *Real time FFT Based SA*

c. *Goertzel based SA*

A. Scan Based SA

A.1 Introduction

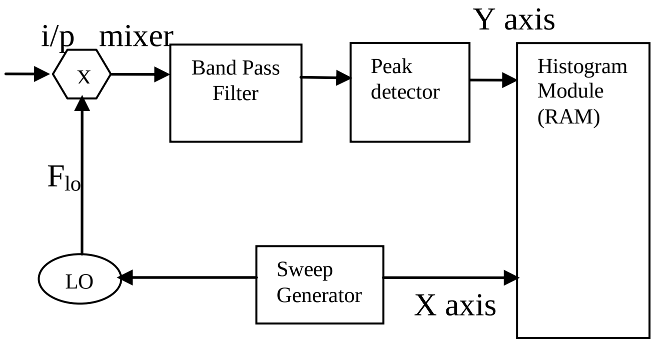
Scan based FFT algorithms are generally software based. Hardware implementation of FFT processor implies huge MAC & storage memory, which are expensive and occupy large chip area.

The main Advantages are fast capture of waveform, able to capture non-repetitive events, able to analyze signal phase.

Disadvantages are frequency limitations, Cost and Speed is less.

B. Real-Time FFT Based SA B.1 Introduction

To find spectrum at real time we go for real time based spectrum analyzers. Real time spectral extraction algorithm is shown in fig.All paragraphs must be indented. All paragraphs must be justified, i.e. both left-justified and right-justified.



B.1. Real time spectrum analyzer

As shown in block diagram it requires Local oscillator, sweep generator, high Q-band pass filter and peak detector. Initially, mixer combines the input test signal and reference signal from the local oscillator. The output modulated signals will have the sum and difference frequency signals. Mixer output will have $f_{LO}+ f_{in}$ and $f_{LO}- f_{in}$ frequencies. The center frequency of BPF is set to the difference frequency. The main function of FIR filter and peak detector is to extract the amplitude component of the input. For narrow bandwidth very high Q- FIR filter with hundreds of taps is required.

Comparison

Modern pulsed systems use advanced waveform and modulation characteristics to obtain greater range resolution, enhanced clutter suppression and superior target recognition. Specifically, performance improvements are achieved using signals with wide bandwidths, low duty-cycles, high linearity and sophisticated modulation. The basic tool for characterizing signals is the spectrum analyzer or signal analyzer, it measures the power content of signals as a function of frequency. Traditional spectrum analyzers use a swept-tuned architecture to achieve high dynamic range and wide frequency measurement ranges.

Advances in digital signal processing have resulted in two significant technological developments: The inclusion of digital IFs in traditional swept-tuned spectrum analyzers, and The emergence of Fast Fourier Transform (FFT)-based analyzers as an alternative to the traditional spectrum analyzer architecture.

The inclusion of digital IFs in traditional spectrum analyzers has greatly enhanced the accuracy, repeatability and speed of these instruments. FFT-based signal analyzers provide unprecedented modulation analysis capability and can result in much faster measurements in some cases, but much slower measurements in other cases.

Selecting the optimal measurement approach requires some knowledge of the instrument capabilities and signal under test. To understand how the instrument's architecture affects the displayed frequency response, measurement speed and dynamic range, the spectral responses of a wideband low duty-cycle radar waveform will be compared using swept-tuned and FFT-based analysis techniques. Some signal analyzers contain both swept and FFT capabilities, allowing a direct comparison of these two techniques within the same instrument. Some signal analyzers can also be used as a vector signal analyzer for measurement of phase profiles, modulation, transient analysis and spectrograms.

FFT-based spectrum analyzer is well for continuous signals but is less effective for pulsed signals due to measurement efficiency. The time required by the analyzer to retune between each segment of the desired spectrum is long relative to the short time data is sampled for each FFT computation. The result is a low probability of intercepting the signal, especially for signals with low duty cycles.

In the swept analyzer, the sweep time was slowed to increase the number of times the pulsed energy was intercepted during the sweep. This approach will not work with an FFT-based spectrum analyzer as it may not even have a sweep time control.

An alternate approach to improving a pulsed RF measurement for an FFT-based spectrum analyzer is to reduce the RBW setting. As the RBW is reduced, the measurement slows, increasing the probability of intercept. If the RBW is reduced enough, missing the signal is no longer an issue since the analyzer sees the spectral components of the signal as continuous waveforms, the sum of which forms the pulsed signal. With a narrow RBW setting, the FFT-based spectrum analyzer actually becomes more efficient at measuring spectrum than a swept analyzer; reducing the RBW to measure a pulsed RF signal has its costs in both measurement speed and dynamic range. For signals within the instrument's analysis bandwidth, an FFT-based signal analyzer can provide rich analysis when implemented as a vector signal analyzer. A vector signal analyzer measures both the magnitude and phase of the signal over time and frequency.

	Scan-based Spectrum Analyzer	FFT-based Spectrum Analyzer
Dynamic Range	Excellent	Excellent
Speed	Fair	Excellent
Modulation Analysis	Optional	Excellent
Transient Analysis	Zero-span	Excellent

Table1. Comparison of Scan and FFT based Analyzers

III. IMPLEMENTATION METHOD

GOERTZEL Algorithm

The proposed method, enables an individual DFT co-efficient to be generated using a simple recursive filter which incorporates a second-order digital resonator, calculate the spectrum at the specific and appropriate frequency from the given input signals and with good frequency resolution. The spectrum analysis using Goertzel Algorithm helps to reduce power and chip area requirement.

The key concept in Goertzel Algorithm is to replace the general FIR filter based spectrum analyzer with fixed center frequency filter for identifying the specified frequency spectral components of a signal.

For a length of N, the Goertzel series is

$$H_k(z) = \frac{1 - W_N^k z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}}$$

Where k=0,1,2,……..,N

And $W_N^k = e^{-2iwpk}$

$$H(z) = \frac{Y(z)}{X(z)} \qquad H(z) = \frac{Y(z)}{S(z)} * \frac{S(z)}{X(z)}$$

The Goertzel algorithm computes a sequence, $s(n)$, given an input sequence, $x(n)$:

$$S(n) = x(n) + 2\cos(2\pi\omega)\, s(n-1) - s(n-2)$$

Where $s(-2) = s(-1) = 0$ and ω is some frequency of interest, in cycles per sample, which should be less than 1/2. This effectively implements a second-order one addition and one subtraction per input sample. For real inputs, these operations are real.

The Z transform of this process is

$$\frac{S(z)}{X(z)} = \frac{1}{1 - 2\cos(2\pi w)\, z^{-1} + z^{-2}} = \frac{1}{(1 - e^{+2\pi iw} z^{-1})(1 - e^{-2\pi iw} z^{-1})}$$

Applying the Fourier transform

$$\frac{Y(z)}{S(z)} = 1 - e^{-2\pi iw} z^{-1}$$

Will give an overall transform of

$$\frac{S(z)}{X(z)}\frac{Y(z)}{X(z)} = \frac{Y(z)}{X(z)} = \frac{1 - e^{-2\pi iw} z^{-1}}{(1 - e^{+2\pi iw} z^{-1})(1 - e^{-2\pi iw} z^{-1})} = \frac{1}{1 - e^{+2\pi iw} z^{-1}}$$

The time-domain equivalent of this overall transform is

$$y(n) = x(n) + e^{+2\pi iw} y(n-1)$$
$$= \sum_{k=\infty}^{n} x(k) e^{+2\pi iw(n-k)} = e^{+2\Pi iwn} \sum_{k=\infty}^{n} x(k) e^{-2\pi iwk}$$

Which becomes, assuming $x(k) = 0$ for all $k < 0$

$$y(n) = e^{+2\pi iwn} \sum_{k=\infty}^{n} x(k) e^{-2\pi iwk}$$

The equation for the $(n + 1)$-sample DFT of x, evaluated for ω and multiplied by the scale factor $e^{+2\pi i\omega n}$.

Applying the additional transform Y (z)/S (z) only requires the last two samples of the s sequence. Consequently, upon processing N samples $x(0)...x(N-1)$, the last two samples from the s sequence can be used to compute the value of a DFT bin, which corresponds to the chosen frequency ω as

$$X(\omega) = y(N-1)e^{-2\pi i\omega(N-1)} = (s(N-1) - e^{-2\pi i\omega} s(N-2))e^{-2\pi i\omega(N-1)}$$

For $\omega N = k$, k- an integer,

This simplifies to

$$X(\omega) = (s(N-1) - e^{-2\pi i\omega} s(N-2))e^{+2\pi i\omega} = e^{+2\pi i\omega} s(N-1) - s(N-2)$$

In either case, the corresponding power can be computed using the same cosine term required to compute s as

$$X(\omega)\, X'(\omega) = s(N-2)^2 + s(N-1)^2 - 2\cos(2\pi\omega)\, s(N-2)\, s(N-1)$$

To compute a single DFT bin for a complex sequence of length N, it requires 2N multiplications and 4N additions/subtractions within the loop, as well as 4 multiplications and 4 additions/subtractions to compute

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X(ω), for a total of 2N+4 multiplications and 4N+4 additions/subtractions (for real sequences, the required operations are half that amount). In contrast, the Fast Fourier transform (FFT) requires 2log2N multiplications and 3log2N dditions/subtractions per DFT bin, but must compute all N bins simultaneously. When the number of desired DFT bins, *M*, is small it is computationally advantageous to implement the Goertzel algorithm, rather than the FFT. Approximately, when

$$M < \frac{5}{6} log_2^N$$

If *N* is not an integral power of 2 in FFT algorithm then zero-padding to the samples would result as,

$$M < \frac{5N_{padded}}{6N} log_2^{(N_{padded})}$$

The Goertzel algorithm can be computed as samples come in. The below figure is the corresponding second order recursive calculation flow. Goertzel algorithm needs only two real multiplications and three real additions to pick up the amplitude of the specified frequency component.

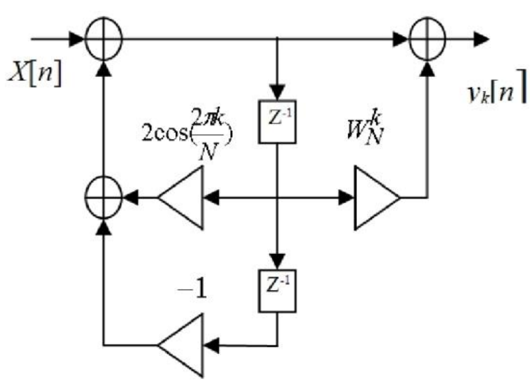


Fig.1. Calculation flow of the second order recursive algorithm

BLOCK DIAGRAM

The figure shows the system function block of a real-time sweep spectral extractor using Goertzel Algorithm

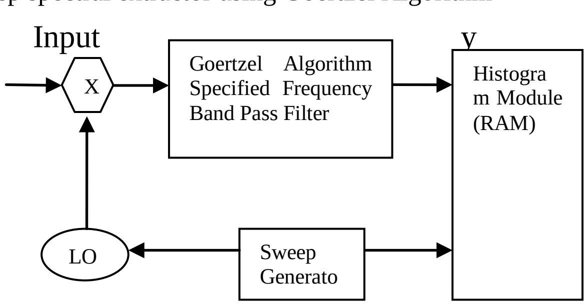
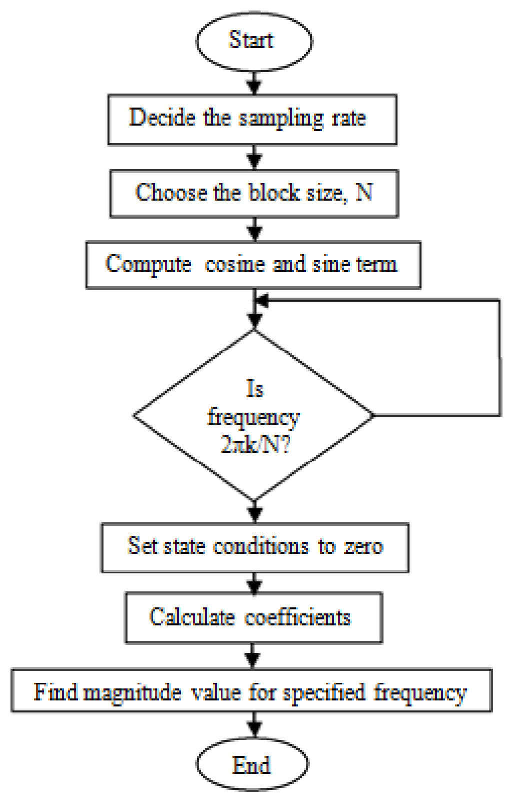


Fig.2. Functional block of a real-time sweep spectral extractor with Goertzel algorithm

The mixer is the key to the success of the analyzer. Mixers are able to operate over a very wide range of signals and offer very low levels of spurious responses. It combines the input test signal and reference signal from the local oscillator. The output modulated signals will have the sum and difference frequency signals. Mixer output will have f_{LO}+ f_{in} and f_{LO}- f_{in} frequencies.The local oscillator within the spectrum analyzer is a key element in the whole operation of the unit. Its performance governs the overall performance parameters of the analyzer. It must be capable of being tuned over a very wide range of frequencies to enable the analyzer to scan over the required range. It must also have a very good phase noise performance. If the oscillator has a poor phase noise performance then it will not be able to make narrowband measurements. The local oscillator generates signal. The local oscillator signal frequency will be always higher than input signal frequency. Generally,

fLO= fin+ fIF

The band pass filter attenuates the unwanted frequencies and allows only required frequency. The band pass filter is set at the difference frequency (f_{IF}) to get the selected frequency component. The filter restricts the bandwidth so that the frequency resolution is effectively increased. The sweep generator drives the sweep of the local oscillator and also the display. The signal from local oscillator will be mixed with the input signal. In this way the horizontal axis of the display is directly linked to the frequency and the histogram will have the information of the frequency for which the spectrum has been analyzed. Histogram module is implemented with RAM. It performs histograming of the spectrum magnitude values with respect to the sweep frequency information and stores the result.



Top Level Module

The top level module gives detail description of the block diagram in fig 3.2. The DDS based LO generates the Numerically Controlled Oscillator (NCO) that will be one of the input signals to the mixer and the other input to mixer is the input test signal. The mixer will combine these two signals and generates a signal which is the difference between these two signals .The output signal of the mixer is at a frequency that the Goertzel algorithm is working and the Goertzel algorithm will identify the frequency components of the signal.

The Goertzel algorithm will generate the frequency components of a signal in terms of the real and imaginary parts that will be applied to the input of the magnitude calculator followed by peak detector which determines the peak of the signal.

The FPGA Spectrum analyzer controller controls all the blocks like frequency bin counter and sample counter. The sample counter counts the samples, when ever this counter reaches the maximum value the frequency bin counter is incremented by one and the sample counter again starts counting until the entire spectrum is covered by the frequency bin counter. The Phase Increment generator generates the corresponding Phase increments for the DDS Core.

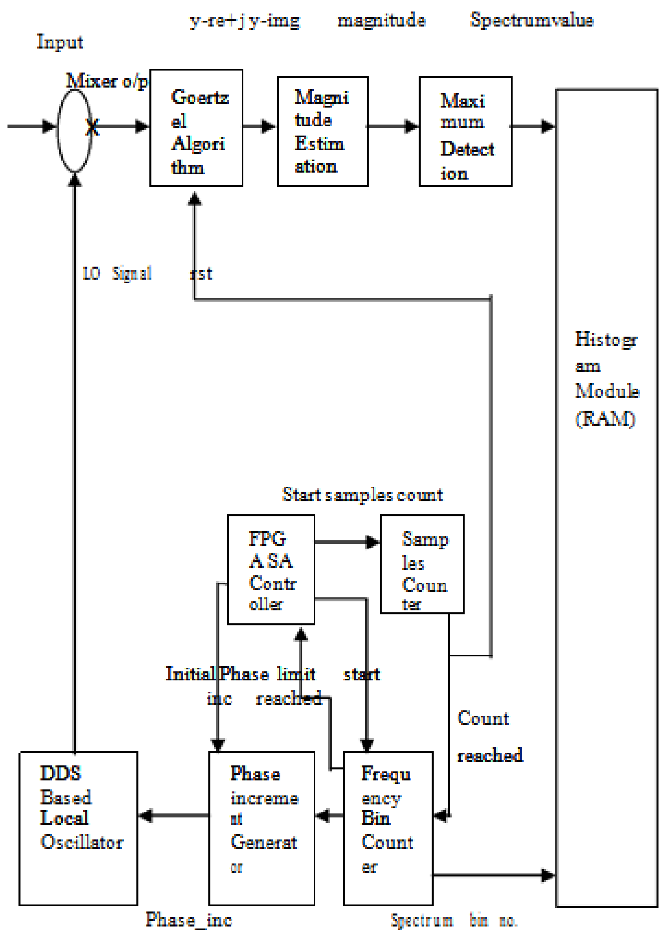


Fig.3. Top level Block Diagram of real time spectral extractor using Goertzel Algorithm

Concurrent Processing Architecture

Introduction

Goertzel Algorithm can be performed parallel for increasing the speed of operation. The block diagram for concurrent processing is as shown below.

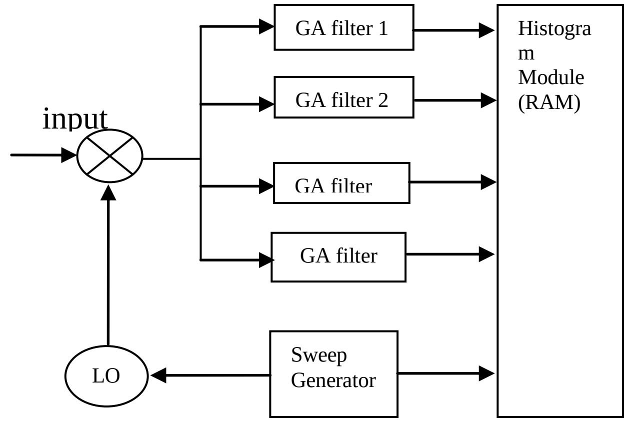


Fig.4. Concurrent processing architecture for sweep spectral extractor

Concurrent processing architecture is used for finding spectrum of different frequencies at same time. The parallel implementation of Goertzel Algorithm increases the performance speed. Each GA filter is set at different frequency for multiple frequency spectrum calculation.

Advantages and Disadvantages

Advantages

1. Parallel processing increases speed.
2. Resolution is more.

Disadvantages

1. Circuit complexity is more.
2. Less area efficient.

IV. RESULTS

In results we compare MATLAB result with VHDL result. The following result is in Chipscope. Here the results are given for 100 KHz and 180 KHz sine waves

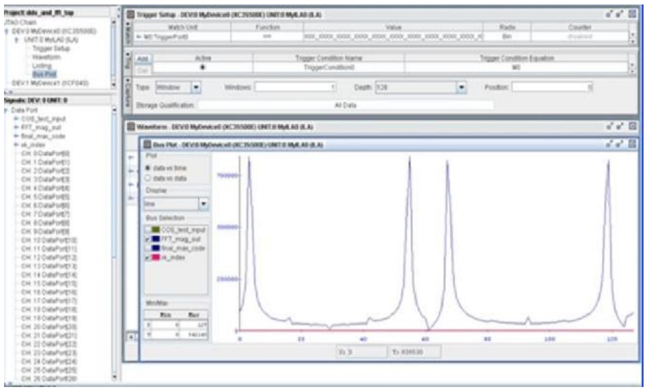


Fig.5. Goertzel algorithm spectrum analyzer result

Matlab Implementation output

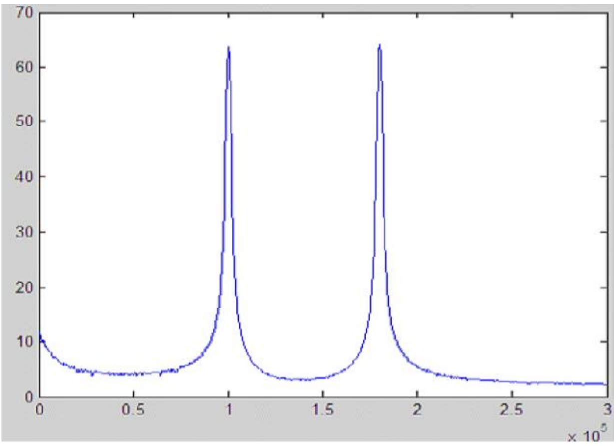


Fig.6. Goertzel Algorithm based spectrum analyzer using MATLAB

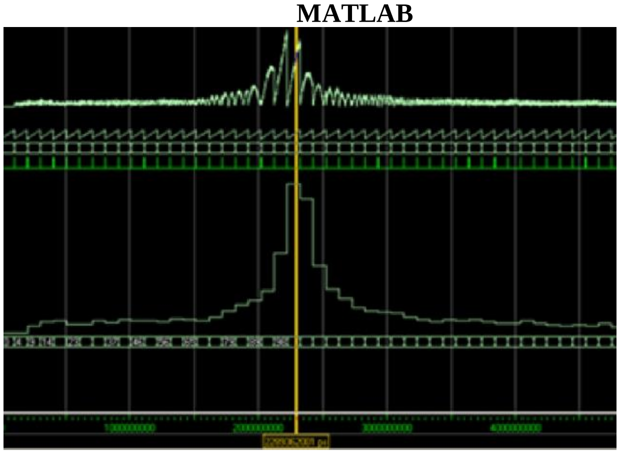


Fig.7. Simulation result of FPGA Top level Module

V. CONCLUSION

The research field of electronics is unlimited in dimensions and the emphasis has been on the evolving trend of the knowledge-based spectrum analysis techniques and their use in signal interception. Spectrum analyzing methods are particularly used for speech, music, biological signals, impulse response of wireless channel and environmental noises. Extraction of a concerned spectrum from sweep of frequencies is a challenging task due to large variations and complexity of the FFT based spectrum analyzer. Currently research is proceeded either heuristically or experimentally to devise spectrum analyzer systems generally applicable to single frequency domains. Goertzel algorithm methods are usually consisting of two main steps: a pre-processing step consisting in required frequency selection (or frequency extraction) and the second step consisting in magnitude of frequency itself. An analyzer which incorporates the spectrum extraction based on Goertzel algorithm with minimization of area and power consumption. Goertzel algorithm for DTMF detection in communication field is proposed. The experimental results shows that the proposed process of using Goertzel algorithm based spectrum analyzer will successfully find the spectrum of a specified frequency with high resolution efficiently

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