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Acoustic Transverse Transmission Loss from ducts with Flat-oval Configuration with Anechoic Termination using Spline Interpolation Coefficients

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Abstract- This paper describes a predictive model for acoustic transverse transmission loss from ducts with flat oval configuration of finite length. The transmission mechanism is essentially that of "mode coupling", whereby higher structural modes in the duct walls get excited because of non-circularity of the wall. Effect of geometry has been taken care of by evaluating Fourier coefficients of the radius of curvature. Emphasis is on understanding the physics of the problem as well as analytical modeling. Effects of the flat ovality, curvature have been demonstrated. With Anechoic termination, progressive approach modeling has been adopted in this paper.

Index Terms- Flat oval ducts, Acoustics, Transverse Transmission loss, anechoic termination, Progressive Wave

I. Introduction

Among the non-circular shells of practical use in automobile industry, after elliptical shells, flat oval shells are of utmost importance. Generally, when space constraints arise under the chassis of automobile for incorporating muffler of circular shape, an elliptical muffler is preferred with minor axis in the vertical direction, so that cross-sectional area is equal to that of the circular shell. But, if that also is unable to match the micro-level space adjustment under the chassis, flat oval shells are preferred that have the advantage of easy manufacturing and adapting to space constraints. Combat Vehicles as well as Heavy Vehicles use this type of configuration if space is a constraint and to reduce the maintenance. In some combat vehicles double flat oval mufflers are used as exhaust ducts. Flat oval shells consist of two semi-circular ducts joined by flat plates as shown below in Figure 1:

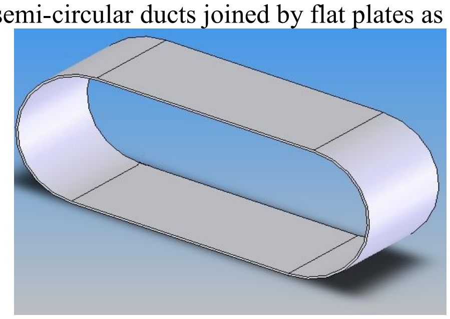
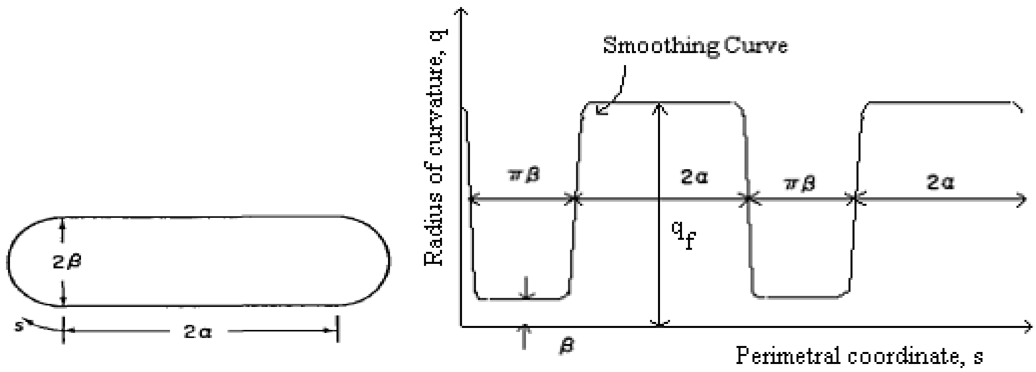


Figure 1: Cross Section of a flat oval duct

However, the Transmission Loss curves for the flat oval ducts are lower than that of ideal circular ducts because of the flat portion. It is known that flat plate radiators are worst of all types of surfaces among the ones that radiate sound.
Cummings [1] analyzed flat oval ducts using finite difference scheme which has its own disadvantages. There is a need for analytical model for prediction of break out noise from flat oval shells.

The approach for finding the Transverse Transmission Loss for Flat Oval duct is same as that of elliptical ducts [2]. However there is a major problem in calculation of Fourier coefficients Am. For elliptical duct, the curvature varies smoothly with angular coordinate 'θ' whereas flat oval ducts don't have continuous curvature in the angular direction.

For a thin non-circular cylindrical shell of constant wall thickness and cross-section as shown in Figure [3], the radius of curvature of the shell is q and this is generally a function of θ, the angular co-ordinate or s, the perimetral co-ordinate, but not of z, the axial co-ordinate.



(a) (b)

Figure 2: (a) Cross-sectional dimensions of a flat-oval duct; (b) Assumed radius of curvature of walls of a flat-oval duct

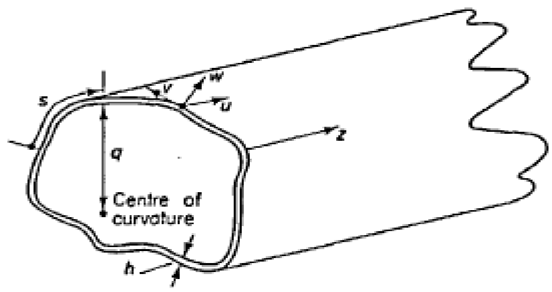


Figure 3: Geometry of a Cylindrical Shell

For a thin non-circular cylindrical shell of constant wall thickness and cross-section as shown in Figure [3], the radius of curvature of the shell is q and this is generally a function of θ, the angular co-ordinate or s, the perimetral co-ordinate, but not of z, the axial co-ordinate. Adopting the Reissner- Naghdi-Berry formulation, governing equations of motion are [3]:

$$\frac{\partial^2 u}{\partial z^2} + \frac{1-\mu}{2}\frac{\partial^2 u}{\partial s^2} + \frac{1+\mu}{2}\frac{\partial^2 v}{\partial z \partial s} + \mu\frac{\partial}{\partial z}\left(\frac{w}{q}\right) - \frac{\ddot{u}}{c_L^2} = 0 \tag{1a}$$

$$\frac{1+\mu}{2}\frac{\partial^2 u}{\partial z \partial s} + \frac{1-\mu}{2}\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{w}{q}\right) - \frac{\ddot{v}}{c_L^2} = 0 \tag{1b}$$

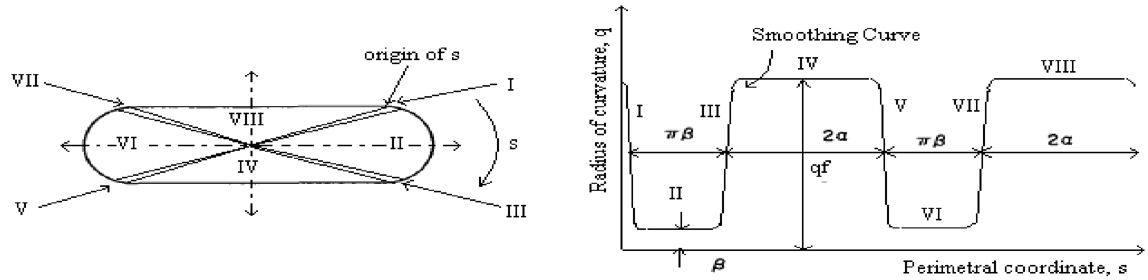
$$\frac{\mu}{q}\frac{\partial u}{\partial z} + \frac{1}{q}\frac{\partial v}{\partial s} + \frac{w}{q^2} + \frac{h^2}{12}\nabla^4 w + \frac{\ddot{w}}{c_L^2} = \frac{p}{h\rho_p c_L^2} \tag{1c}$$

where u, v and w are the wall displacement components in the axial, azimuthal and radial directions respectively, c_L, ρ_p, and μ are (respectively) the longitudinal wave speed in, and the density and Poisson's ratio of, the shell material, h is the shell thickness, a is the average radius of the shell, 'p' is the pressure difference (internal minus external pressure) across the shell (generally a function of s and z) and $\nabla^2 = \partial^2/\partial z^2 + \partial^2/\partial s^2$.

In Eq. (1), the radius of curvature q, of the shell, as a function of the perimetral co-ordinate s (See Figure 3--u, v and w are the orthogonal wall displacement components), and its first, second and third derivatives with respect to s, appear. This could cause difficulty in solving the equations of motion for a flat-oval duct, because q is constant over the curved duct walls, but tends to infinity on the flat walls as shown in Figure 2(a). Thus q, and of course its derivatives, are discontinuous. To overcome this problem, q is made very large, but finite, on the flat walls and a half-sinusoid "smoothing function" is used to give a smooth variation in q(s), q|, q|| and q||| between flat and curved walls. Only a subtle alteration of the actual geometry (which has no significant effect on the structural motion) is thereby created, but the solution of the equations of motion is facilitated. Figure 2(a) shows the characteristic dimensions of the duct cross-section and Figure 2(b) shows the assumed variation of q with s; here, q_f is the radius of curvature of the flat walls which is assumed to be very large compared to β.

II. Fourier Coefficients of Flat Oval Duct

To find the Fourier coefficients A_m, expressions for radius of curvature (q) throughout the perimeter must be known. So at the region of discontinuity radius of curvature is assumed to be SPLINE polynomial of third degree, so that even third order derivative with respect to perimetral coordinate exists.



(a) (b)

Figure 4: a) Perimetral division of flat oval duct into 8 parts, b) Representation of the radius of curvature with respect to perimetral coordinate

Let us adopt ζ (zeta), a very the small number (of the order 10-3), to represent the perimetral divisions in the 8 parts shown in the Figure 4. These may be represented in terms of the perimetral coordinate‘s’ as follows.

I at $s = 0$, $q = q_f$ and at $s = (2\alpha+\pi\beta)\,\zeta$, $q = \beta$
II at $s = (2\alpha+\pi\beta)\,\zeta$, $q = \beta$ and at $s = 2\alpha\zeta+\pi\beta\,(1-\zeta)$, $q = \beta$
III at $s = 2\alpha\zeta+\pi\beta\,(1-\zeta)$, $q = \beta$ and at $s = 2\alpha\zeta+\pi\beta+2\alpha\zeta$, $q = q_f$
IV at $s = 2\alpha\zeta+\pi\beta+2\alpha\zeta$, $q = q_f$ and at $s = 2\alpha\zeta+\pi\beta+2\alpha\,(1-\zeta)$, $q = q_f$
V at $s = 2\alpha\zeta+\pi\beta+2\alpha\,(1-\zeta)$, $q = q_f$ and at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\zeta$, $q = \beta$
VI at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\zeta$, $q = \beta$ and at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\,(1-\zeta)$, $q = \beta$
VII at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\,(1-\zeta)$, $q = \beta$ and at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta+2\alpha\zeta$, $q= q_f$
VIII at $s = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta+2\alpha\zeta$, $q = q_f$ and at $s = 2\pi\beta+4\alpha$, $q = q_f$

Defining‘s’ points in all regions as I: s1 to s2; II: s2 to s3; III: s3 to s4; IV: s4 to s5; V: s5 to s6; VI: s6 to s7; VII: s7 to s8; VIII: s8 to S,

$s1 = 0$
$s2 = (2\alpha+\pi\beta)\,\zeta$
$s3 = 2\alpha\zeta+\pi\beta\,(1-\zeta)$,
$s4 = 2\alpha\zeta+\pi\beta+2\alpha\zeta$
$s5 = 2\alpha\zeta+\pi\beta+2\alpha\,(1-\zeta)$
$s6 = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\zeta$
$s7 = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta\,(1-\zeta)$
$s8 = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta+2\alpha\zeta$
$S = 2\alpha\zeta+\pi\beta+2\alpha+\pi\beta+2\alpha\,(1-\zeta) = 2\pi\beta+4\alpha$

So, from the above values we find the SPLINE interpolation polynomial for regions I, III, V and VII. In region II, $q= \beta$; in region IV, $q= q_f$; in region VI, $q= \beta$ and in region VIII, $q= q_f$.

Denoting β=beta; ζ=zeta; α/β=delta; q_f =cu*beta (cu is of order 25) and π=pi, the SPLINE interpolating polynomials for regions I, III, V and VII are being obtained using MAPLE-Symbolic Package.

Ratio of radius of curvature square to average radius square in terms of Fourier coefficients as

$$\frac{q_f^{\,2}}{average\ \ radius^{\,2}} = \sum_{m=0}^{\infty} A_m \cos\left(\frac{2\pi\ m\ s}{S}\right) \tag{2}$$

If q_x is the radius of curvature in the region ‘x’ (x=I to VIII), then average radius can be found for the flat oval duct as avr_f (flat oval)=√(Area of duct/π), Area of duct (A_r) = $\pi\ \beta2 + 4\ \alpha\ \beta$, Perimeter of duct (S) = $2\ \pi\ \beta + 4\ \alpha$, costerm = cos(2* π*m*s/S) (where ‘m’ is mode number), s1 to s8 and S are defined already.

A_m is found for the given mode along the perimeter (S) for i^{th} region as follows:

In region x, $A_{mx} = \frac{1}{S}\int_{s_x}^{s_{i+1}} \frac{q_x^2}{avr_f^2} \cos term \; ds \quad (x = I,\ II,\ ...,\ VIII).$ (3 a, b, c, d, e, f, g, h)

So, finally $A_m = A_mI + A_mII + A_mIII + A_mIV + A_mV + A_mVI + A_mVII + A_mVIII$ (4)

III. Analytical Studies using Progressive Wave Approach (Anechoic Termination)

With the end termination as anechoic i.e., Reflection Coefficient R~0 which generates a progressive wave pattern in the flat oval duct. For obtaining Radial Wall Displacement W_m same procedure as mentioned in Ref [3] of Eq.34 is adopted.
The wall displacement components are written in terms of positive traveling waves only,

$$u = U(s)e^{i(\omega t - k_z z)} \quad , v = V(s)e^{i(\omega t - k_z z)} \, , w = W(s)e^{i(\omega t - k_z z)} \tag{5 a-c}$$

(k_z being the axial wave number)

$$U(s) = \sum_{m=0}^{\infty} U_m \cos\left(\frac{2\pi\ m\ s}{S}\right), V(s) = \sum_{m=0}^{\infty} V_m \sin\left(\frac{2\pi\ m\ s}{S}\right) \tag{6 a, b}$$

$$W(s) = \sum_{m=0}^{\infty} W_m \cos\left(\frac{2\pi\ m\ s}{S}\right) \tag{6 c}$$

The acoustic pressure within the duct may be written as

$$p = P\ e^{i(\omega t - k_z z)} \tag{7}$$

Substituting equations (5) to (7) in the Eq. (1) and assuming without plane strain approximation, we get

$$\left[\frac{\omega^2}{c_L^2} - k_z^2 - \left(\frac{1-\mu}{2}\right)\left(\frac{2\pi\ m}{S}\right)^2\right] U_m - ik_z\left(\frac{1+\mu}{2}\right)\left(\frac{2\pi\ m}{S}\right) V_m - \frac{ik_z \mu}{avr_f} W_m = 0 \tag{8a}$$

$$ik_z\left(\frac{1+\mu}{2}\right)\left(\frac{2\pi\ m}{S}\right) U_m + \left[\frac{\omega^2}{c_L^2} - \left(\frac{2\pi\ m}{S}\right)^2 - \left(\frac{1-\mu}{2}\right) k_z^2\right] V_m - \left(\frac{1}{avr_f}\right)\left(\frac{2\pi\ m}{S}\right) W_m = 0 \tag{8b}$$

$$\left[- ik_z (avr_f)\mu\right] U_m + avr_f \left(\frac{2\pi\ m}{S}\right) V_m + \left[1 - \frac{\omega^2 avr_f^{\ 2}}{c_L^2}\right] W_m = \left(P\ avr_f^{\ 2} / h\rho_p c_L^2\right) A_m \tag{8c}$$

In the above equations it is assumed that at differential scale 'q' is constant and equals to average radius of flat oval duct 'avr_f', so $q^|$ is zero.

In the present context k_z=k_o. On further simplification using equations (8(a)), (8(b)) and (8(c)), W_m can be expressed in terms of A_m as:

$$W_m = P\ avr_f^{\ 2} A_m \left(h\rho_p c_L^2 \left\{ - ik_o (avr_f)\mu \frac{U_m}{W_m} + avr_f \left(\frac{2\pi\ m}{S}\right)\frac{V_m}{W_m} + \left[1 - \frac{\omega^2 avr_f^{\ 2}}{c_L^2}\right] \right\} \right)^{-1} \tag{8d}$$

Substituting A_m from Eq. (4) in the above Eq. (8(d)) we can evaluate the radial wall displacement W_m.
For obtaining radiation from the duct walls the procedure is same as in Ref [4]. And Transverse Transmission Loss can be evaluated by means of equations (9) and (10), and incorporating cubic spline functions for calculating Fourier Coefficients A_m.
The internal sound power is calculated as

$$\Pi_i = \frac{\pi\ a^2 P^2}{2\rho_o c_o} \tag{9}$$

(Where *P* is the unit internal sound pressure amplitude). The sound transmission loss of the duct wall is defined as the logarithmic ratio between the internal sound power per unit cross-sectional area and the radiated sound power per unit surface area and is given by

$$TTL = 10\log\left(\frac{2L\Pi_i}{a\Pi_t}\right) \quad (10)$$

IV. Results and Discussion

Based on the approach given in the section 3, a code is developed in MATLAB for the flat oval shell. Transverse Transmission loss vs. Frequency curves are plotted for the flat oval duct and ideal circular duct, i.e., DUCT I of reference [4]. Figure 5 show that the flat oval shell would radiate much more breakout noise than the equivalent circular shell. Average radius of duct is taken same as DUCT I, i.e., 0.178 m; α as 0.0255 m and β as 0.1625 m (From Figure 2(a)).

Data of h, a, ρ_p, E, η, μ specify the rest of the "system parameters" for Duct I without loss of generality. Values are h=1.22 mm, a=0.178 m, ρ_p=7800 kg/m3, E=210 GPa, η =0.1, μ=0.3.

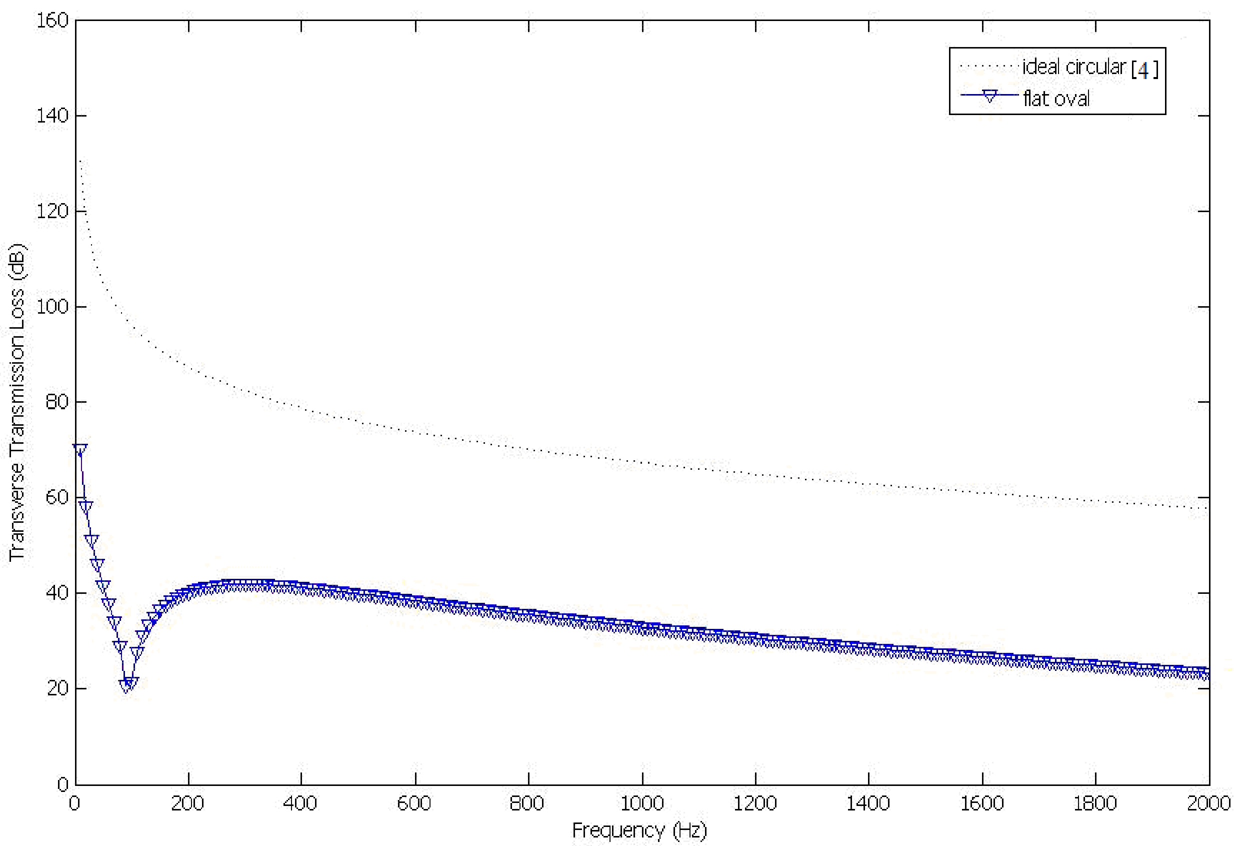


Figure 5: Comparison of the flat oval duct with an equivalent ideal circular duct using progressive wave approach

Comparison with experimental results from reference [1] (DUCT II) is done here and experimental results which were measured at one-third octave band frequency are shown in Figure 6.

Experimental estimation of transverse transmission Loss has been obtained from Ref [1]. Analytical predictions are following the same pattern as obtained from the experiments. Some deviation from experimental results can be observed at lower frequencies. This may be due to the effect of localized flattening at the seam weld in the experimental duct. Impact of flat portion on the acoustic transverse transmission loss has been confirmed with this analytical evaluation. In general the physics of the problem has been understood and corroborated with Ref [1, 4].

Flat-oval Configuration of ducts which are of generic type when used in specific space optimization industrial equipment for exhaust systems can be verified with the results presented here. The spline interpolation assumption is a novel approach for the analytical modeling. An experimental study for every configuration seems to be tedious and time consuming. An analytical study not only gives a feel of the physics of the problem but also reduces time of design considerably.

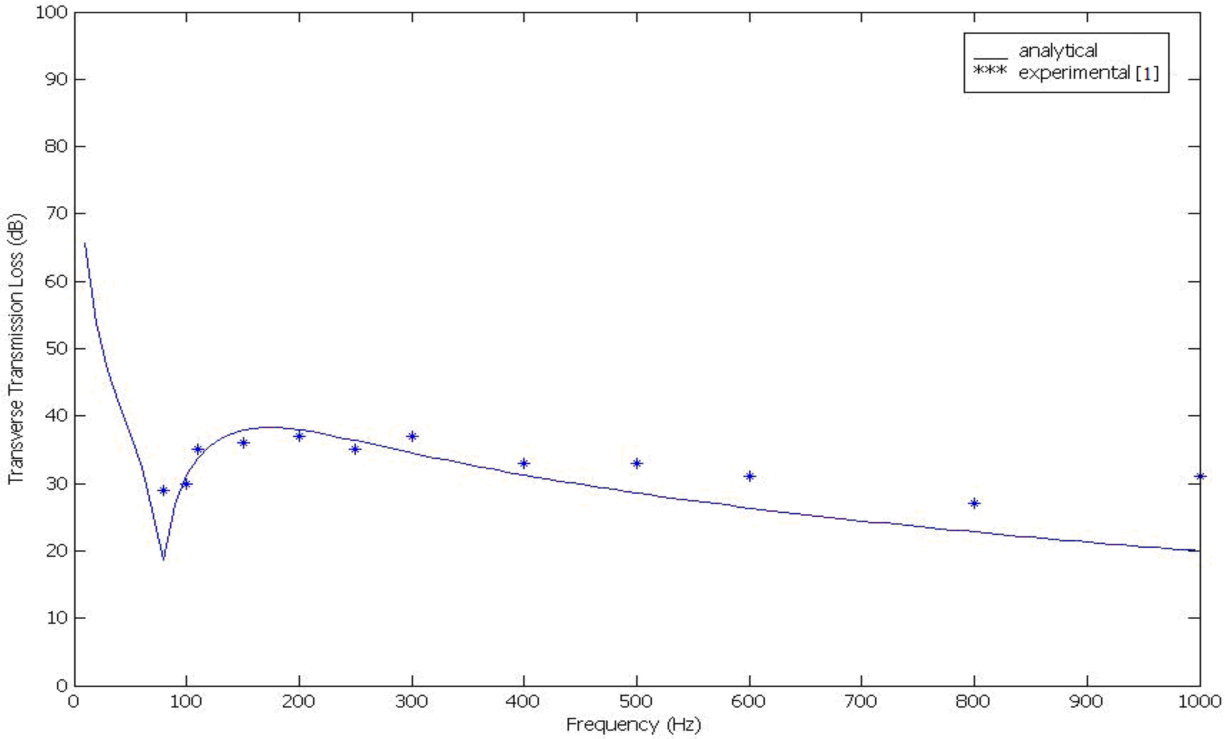


Figure 6: Comparison between analytical and experimental results for the flat oval duct using the progressive wave approach-anechoic termination

With the variation in length of the shell effect of transverse transmission loss had been plotted in Figure 7. From Figure 7 it is observed that for frequencies up to 1000 Hz, 20 m duct has higher TTL, while above this frequency, TTL is almost same. This is due to the fact that up to 1000 Hz the higher structural modes tend to radiate sound power principally from the ends of the duct, because of scattering effects. This means that the radiated power per unit length will decrease as the duct length increases.

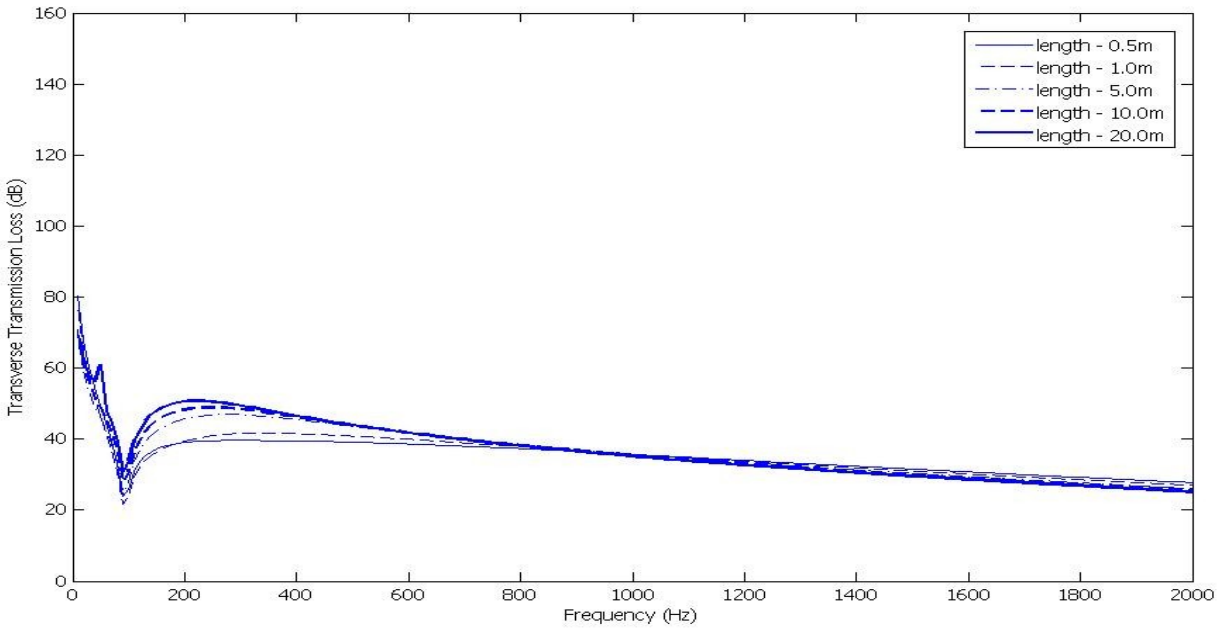


Figure 7: Effect of the length on prediction of transverse transmission loss for flat oval duct with anechoic termination

Comparison is made with flat oval duct, rectangular duct, ideal circular duct and elliptical duct. Here, equivalent radius of ideal circular duct, elliptical duct, flat oval duct and rectangular duct is assumed to be same. Length of the ideal circular duct, elliptical duct, flat oval duct and rectangular duct is the same i.e., 1.17 meters (rectangular duct dimensions) as in reference [5]. Ovality of elliptical duct is taken as 0.5.

Rectangular duct dimensions:

Width =0.206 m; Height =0.258; Length of duct =1.17 m; thickness=1.295 mm; Young's Modulus =1.816 E11 N/m2, density of shell=8278 Kg/ m3 ; poisons ratio =0.291 and loss factor=0.01

Using the above Rectangular duct dimensions equivalent dimension of ideal circular duct is obtained as 0.13 meters which is the radius of the duct.

Using the above Rectangular duct dimensions equivalent dimension of elliptical duct is obtained as 0.1839 meters which is the length of major axis and 0.0919 meters as length of minor axis with ovality 0.5.

Using the above Rectangular duct dimensions equivalent dimension of flat oval duct is obtained as 0.0187(α) meters which is the length flat portion and 0.1187(β) meters as the radius of the circular portion.

Comparison is made for anechoic termination which is shown in Figure 8 and it is observed that for anechoic termination TTL of ideal circular duct is the highest, TTL of elliptical duct lies between ideal circular duct and flat oval duct and TTL of rectangular duct is the least.

For anechoic end termination Transverse Transmission Loss for flat oval duct with different delta values is plotted in Figure 9. Here delta is the ratio of alpha to beta (delta= α/β). With an increase in this value delta (ovality of flat oval duct) there is an increase in the radiated power from the flat portion of flat oval duct which signifies a decrease in Transverse transmission Loss. It can be observed from Figure 9 that even a little ovality (delta = 0.1) of flat oval duct results in a large decrease in TTL with respect to the ideal circular duct. Interestingly, further increase in ovality (delta = 0.5) results in only a little additional deterioration, and the resulting TTL remains significantly higher than that of an equivalent rectangular duct which has comparatively little transverse stiffness.

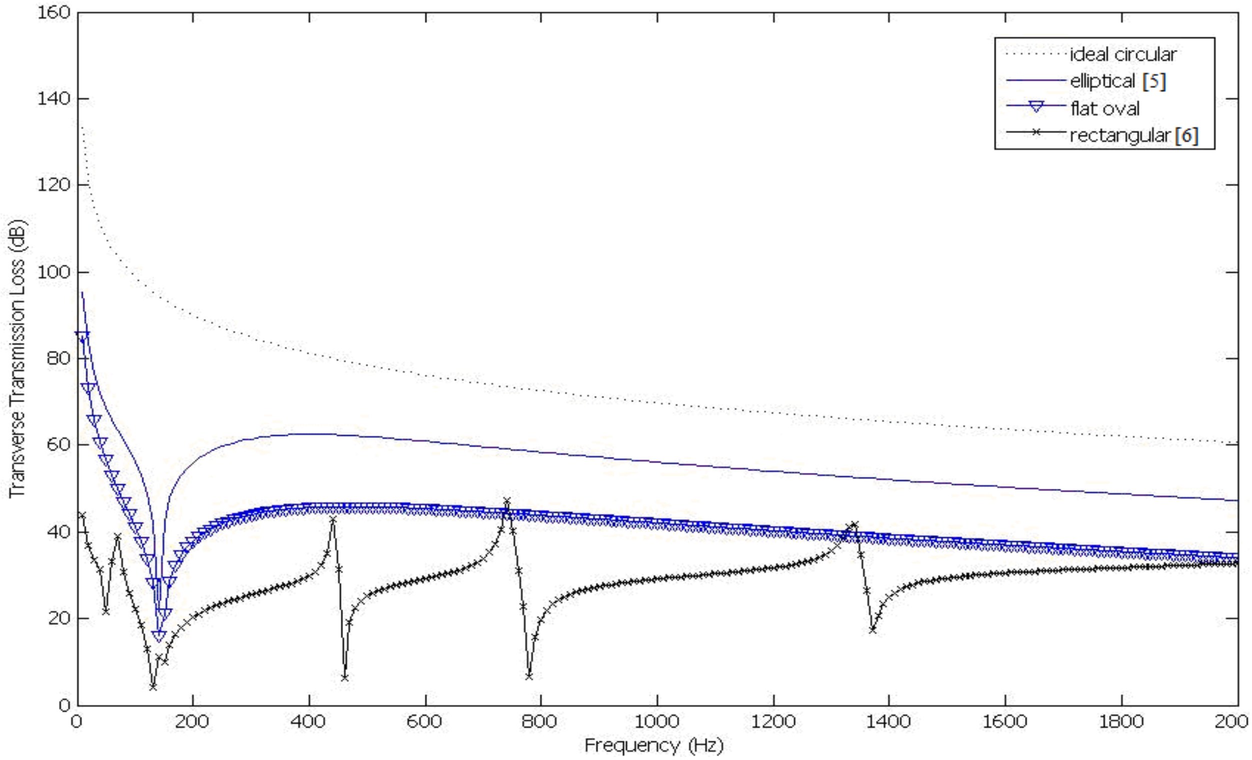


Figure 8: Comparison between ideal circular, elliptical, flat oval and rectangular duct with anechoic termination

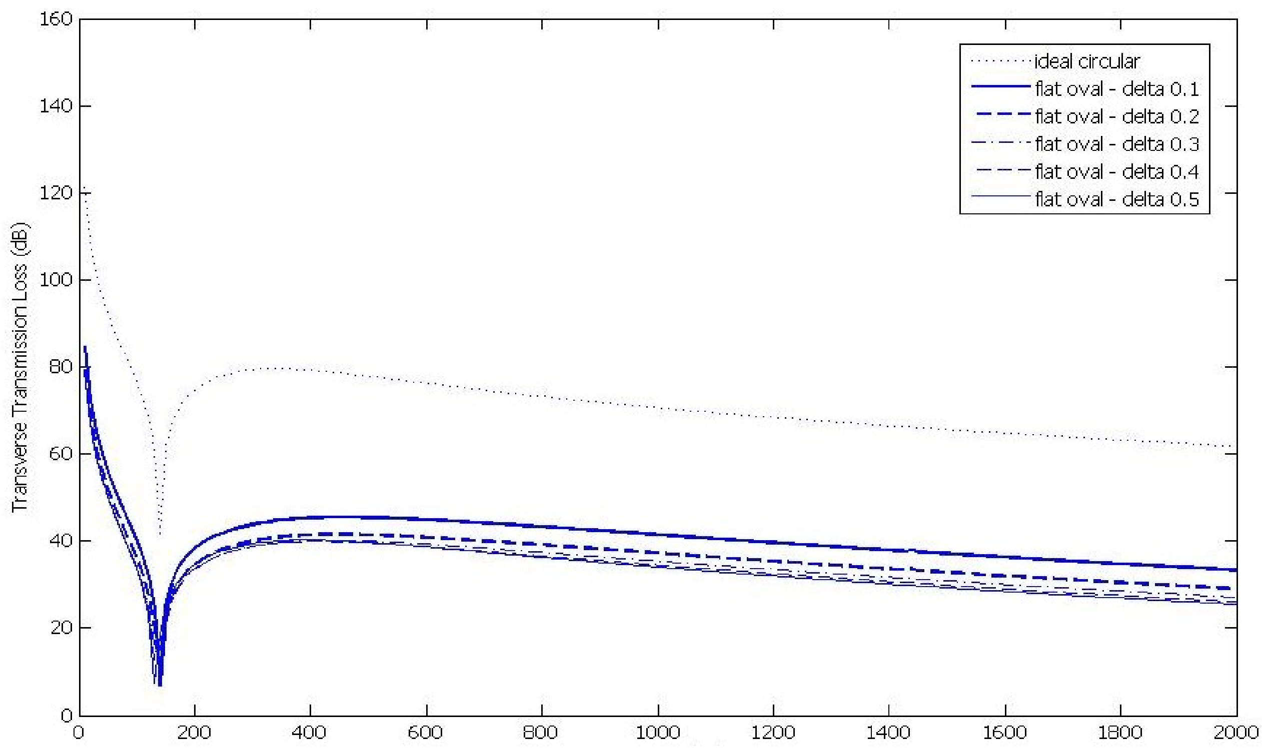


Figure 9: Effect of delta (α/β) value on the transverse transmission loss for flat oval duct with anechoic termination

V. CONCLUSION

The analytical model has been formulated successfully for a flat oval duct with anechoic termination. Progressive approach acoustic transverse transmission loss has been obtained for flat oval configuration. Results found are well in good agreement with published results of Ref [1]. First Resonance has been observed for flat oval ducts at much lower frequencies in comparison with the circular ducts. With this approach engineers can evaluate the feasibility of using the flat oval ducts instead of circular ducts where space optimization is important.

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