AN ALGORITHMIC APPROACH TO SOLVE TRANSPORTATION PROBLEMS WITH THE AVERAGE TOTAL OPPORTUNITY COST METHOD

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Abstract: The current new algorithmic approach to solve the Transportation Problem (TP) is based upon the Total Opportunity Cost (TOC) of a Transportation Table (TT). Opportunity cost in each cell along each row of a TT is the difference of the corresponding cell value from the lowest cell value of the corresponding row. Similarly, opportunity cost in each cell along each column of a TT is the difference of the corresponding cell value from the lowest cell value of the corresponding column. Total Opportunity Cost Table (TOCT) is formed by adding the opportunity cost in each cell along each row and the opportunity cost in each cell along each column and putting the summation value in the corresponding cell. The current algorithm considers the average of total opportunity costs of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of total opportunity costs of cells along each column identified as Column Average Total Opportunity Cost (CATOC). Allocations of costs are started in the cell along the row or column which has the highest RATOCs or CATOCs. The Initial Basic Feasible Solution (IBFS) obtained by the current method is better than some other familiar methods which is discussed in this article.

Keywords: TP, TOC, TT, TOCT, RATOC, CATOC

I. INTRODUCTION

Transportation cost has significant impact on the cost and the pricing of raw materials and goods. Supplier and producer try to control the cost of transportation. The way how the desirable transportation cost can be obtained is the subject matter of transportation problems in linear programming. Some conventional methods to find the minimum transportation cost are North West Corner (NWC) method, Matrix Minima method/ Least Cost method, Row Minima method, Column Minima method, and Vogel's Approximation Method (VAM). Matrix Minima method and VAM are considered to provide the better IBFS. Besides the covenantal methods many researchers has provide many methods to find a better IBFS of a TP. Some of the important related works the current research has deal with are: 'A New Approach for Solving Transportation Problems with Mixed Constraints' [1] by P. Pandian and G.Natarajan; N. M. Deshmukh's 'An Innovative Method for Solving Transportation Problem' [2]; 'Modified Vogel's Approximation Method for Unbalance Transportation Problem' [3] by N. Balakrishnan; Serder Korukoglu and Serkan Balli's 'An Improved Vogel's Approximation Method (IVAM) for the Transportation Problem' [4]; Harvey H. Shore's 'The Transportation Problem and the Vogel's Approximation Method' [5]; 'A modification of Vogel's Approximation Method through the use of Heuristics' [6] by D.G. Shimshak, J.A. Kaslik and T.D. Barelay; A. R. Khan's **'**A Re-solution of the Transportation Problem: An Algorithmic Approach' [7]; 'A new approach for finding an Optimal Solution for Transportation Problems' by V.J. Sudhakar, N. Arunnsankar, and T. Karpagam [8]. Md. Amirul Islam *et al*. [9] calculate the Difference Indicators by taking the difference of the largest and the next largest cell value of each row and each column of the TOCT for the allocation of units of the TT. The cited algorithms in this article are beneficial to find the IBFS to solve transportation problems. Also, the current research presents a useful algorithm which gives a better IBFS in this connection.

II. ALGORITHM

The developed algorithm in the current research involves two parts:

- ❖ Algorithm for Total Opportunity Cost Table (TOCT)
- ❖ Algorithm for transportation allocation

Algorithm for TOCT

Step 1 Subtract the smallest entry from each of the elements of every row of the TT and place them on the right-top of corresponding elements.

Step 2 Apply the same operation on each of the columns and place them on the right-bottom of the corresponding elements.

Step 3 Form the TOCT whose entries are the summation of right-top and right-bottom elements of Steps 1 and 2.

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Algorithm for Allocation

Step 1 Place the average of total opportunity costs of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of total opportunity costs of cells along each column identified as Column Average Total Opportunity Cost (CATOC) just after and below the supply and demand amount respectively within first brackets.

Step 2 Identify the highest element among the RATOCs and CATOCs, if there are two or more highest elements; choose the highest element along which the smallest cost element is present. If there are two or more smallest elements, choose any one of them arbitrarily.

Step 3 Allocate x_{ij} = min (a_i , b_j) on the left top of the smallest entry in the (i, j) th of the TT.

Step 4 If $a_i < b_j$, leave the ith row and readjust b_j as $b_j^{/} = b_j - a_i$.

If $a_i > b_j$, leave the jth column and readjust a_i as $a_i^{/} = a_i - b_j$.

If $a_i = b_j$, leave either ith row or j-th column but not both.

Step 5 Repeat Steps 1 to 4 until the rim requirement satisfied.

Step 6 Calculate $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$, z being the minimum transportation cost and c_{ij} are the cost elements of the TT.

III. NUMERICAL ILLUSTRATIONS

Illustration 01

The per unit transportation cost (in thousand dollar) and the supply and demand (in number) of motor bikes of different factories and showrooms are given in the following transportation table.

Factories	Showrooms				Supply (a_i)
	D_1	D_2	D_3	D_4	
W_1	9	8	5	7	12
W_2	4	6	8	7	14
W_3	5	8	9	5	16
Demand (b_j)	8	18	13	3	42

Table: 1.1

We want to solve the transportation problem by the current algorithm.

Solution

The row differences and column differences are:

Factories	Showrooms				Supply
	D_1	D_2	D_3	D_4	
W_1	9_5^4	8_2^3	5_0^0	7_2^2	12
W_2	4_0^0	6_0^2	8_3^4	7_2^3	14
W_3	5_1^0	8_2^3	9_4^4	5_0^0	16
Demand	8	18	13	3	42

Table: 1.2

Therefore, the TOCT is:

Factories	Showrooms				Supply
	D_1	D_2	D_3	D_4	
W_1	9	5	0	4	12

W_2	0	2	7	5	14
W_3	1	5	8	0	16
Demand	8	18	13	3	42

Table: 1.3

The allocations with the help of RATOCs and CATOCs are:

Factories	Showrooms				Supply	RATOC			
	D_1	D_2	D_3	D_4					
W_1	9	5	$^{12}0$	4	12	(4.5)	-	-	-
W_2	0	$^{13}2$	$^{1}7$	5	14	(3.5)	(3.5)	(2.3)	-
W_3	$^{8}1$	$^{5}5$	8	$^{3}0$	16	(3.5)	(3.5)	(2)	(2)
Demand	8	18	13	3	42				
CATOC	(3.3)	(4)	(5)	(3)					
	(0.5)	(3.5)	(7.5)	(2.5)					
	(0.5)	(3.5)	-	(2.5)					
	-	-	-	-					

Table: 1.4

Therefore, the allocations in the original TT are:

Factories	Showrooms				Supply
	D_1	D_2	D_3	D_4	
W_1	9	8	$^{12}5$	7	12
W_2	4	$^{13}6$	$^{1}8$	7	14
W_3	$^{8}5$	$^{5}8$	9	$^{3}5$	16
Demand	8	18	13	3	42

Table: 1.5

The transportation cost is $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

$$z = 5\times12 + 6\times13 + 8\times1 + 5\times8 + 8\times5 + 5\times3$$

= 241 $

Illustration 02

A company manufactures toilet tissues and it has three factories S_1, S_2 and S_3 whose weekly production capacities are 9, 8 and 10 thousand pieces of toilet tissues respectively. The company supplies tissues to its three showrooms located at D_1, D_2 and D_3 whose weekly demands are 7, 12 and 8 thousand pieces respectively. The transportation costs per thousand pieces are given in the next Transportation Table:

Factories	Showrooms			Supply(a_i)
	D_1	D_2	D_3	
S_1	3	3	5	9
S_2	6	5	4	8
S_3	6	10	7	10
Demand(b_j)	7	12	8	27

Table: 2.1

Solution:

The row differences and column differences are:

Factories	Showrooms			Supply
	D_1	D_2	D_3	
S_1	3^{0}_{0}	3^{0}_{0}	5^{2}_{1}	9
S_2	6^{2}_{3}	5^{1}_{2}	4^{0}_{0}	8
S_3	6^{0}_{3}	10^{4}_{7}	7^{1}_{3}	10
Demand	7	12	8	27

Table: 2.2

Therefore, the TOCT is:

Factories	Showrooms			Supply
	D_1	D_2	D_3	
S_1	0	0	3	9
S_2	5	3	0	8
S_3	3	11	4	10
Demand	7	12	8	27

Table: 2.3

The allocations with the help of RATOCs and CATOCs are:

Factories	Showrooms			Supply	RATOC		
	D_1	D_2	D_3				
S_1	0	$^{9}0$	3	9	(1)	(1.5)	(1.5)
S_2	5	$^{3}3$	$^{5}0$	8	(2.6)	(1.5)	(1.5)
S_3	$^{7}3$	11	$^{3}4$	10	(6)	(7.5)	-
Demand	7	12	8	27			
CATOC	(2.6)	(4.6)	(2.3)				
	-	(4.6)	(2.3)				
	-	(1.5)	(1.5)				

Table: 2.4

Therefore, the allocations in the original TT are:

Factories	Showrooms			Supply
	D_1	D_2	D_3	
S_1	3	$^{9}3$	5	9
S_2	6	$^{3}5$	$^{5}4$	8
S_3	$^{7}6$	10	$^{3}7$	10
Demand	7	12	8	27

Table: 2.5

The transportation cost is $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

$$z = 3\times 9 + 5\times 3 + 4\times 5 + 6\times 7 + 7\times 3$$

= 125 units

IV. COMPARISON OF RESULTS

The current method yields a better IBFS of a TP then other conventional methods. To show this, a comparison is presented in the following table:

Methods	Solutions

	Illustration – 1	Illustration – 2
Current Method	241	125
North-West Corner Method	320	143
Matrix Minima Method	248	159
VAM	248	143
Optimal Solution	240	125

Table: 3

V. CONCLUSION

The current method considers averages of total cost along each row and each column which is totally new concept. The solution obtained by the current method is near optimal or optimal. The developed method is effective for both the large and small size TP.

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