Tests for Equality of Coefficients of Variation of Two Normal Distributions for Correlated Samples

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Abstract: Coefficient of variation (C.V) is widely used as a measure of dispersion in applied research. C.V is unit less and thus facilitates the comparison of variability in two or more groups. Several tests have been proposed in the past for testing equality of C.Vs of two independent normal distributions. In plant sciences, medical sciences several characteristics of the plant or the subjects are to be compared regarding the variability and the samples are correlated. In this paper six tests are proposed for testing equality of C.Vs of a Bivariate normal distribution. The asymptotic null distribution of the entire test statistic is Chi-square distribution with 1 degree of freedom. The adequacy of the Chi-square approximation for finite samples is examined using simulation.

Keywords:Coefficient of variation (C.V); Normal distribution; Bivariate normal distribution; Chi-square approximation; Simulation.

I. INTRODUCTION

Coefficient of Variation (C.V) is widely used as a measure of variation by the researchers in the applied disciplines like finance, climatology, engineering etc. The popularity of C.V stems from the fact that it is unit less and can be interpreted easily than standard deviation. C.V interpreted as relative risk in the area of finance [1]. In stock market analysis, it is interpreted as volatility per mean return and inverse C.V is referred as Sharp ratio [1].

Historically the first research work on C.V dates back to 1932 [2]. Initially, the researchers were interested to develop improved Confidence Interval for the C.V of the Normal Distribution. Resent references in these directions are Banik and Kibria [3] and see the references cited in this paper for earlier works.

Although 100(1-α)% confidence interval at a level α test are interrelated, a formal Likelihood Ratio(LR) Test for equality of C.Vs of independent Normal Distributions was first introduced by Bennett [4] using modified C.V. Shafer and Sullivan [5] improved these tests using conditional likelihood. LR Test for equality of C.Vs of two independent Normal Distributions was proposed by Miller and Karson [1]. Doornbos and Dijkstra [6] extended the LR test for testing the equality of C.Vs of more than two independent Normal Distributions. Rao and Bhatta [7] proposed Wald test (also see [8]) for the same hypothesis. Gupta and Ma [9] proposed Score test for testing the equality of C.Vs. of two or more Normal Distributions.

Following the generalized variable approach Tsui and Weerahandi [10], Jafari and Behboodian[11]developed generalized test statistic for testing the common C.V of two or more independent Normal Distributions. The LR, Wald and Score tests and their perturbed version were not robust against the assumption of normality. This has motivated Cabras, Mostallino and Racugno [12] to propose bootstrap tests for equality of C.Vs of two distributions. All these tests assumed that the samples are independent. In practice, correlated samples are often encountered. In medical studies many of the periodical characters are interrelated. For example, in the field of Anatomy, when gender has to be decided using the various measurements of the skull, these measurements are interrelated. This example is discussed in section 4. In the stock market the stock prices of various scripts are related and testing for equality of volatility for mean return for two or more scripts, the correlation needs to be accounted for Singh [13] proposed generalized test for testing equality of C.Vs of p variates Normal Distributions. This test is computationally tedious and it is not appealing to the scientists in the applied disciplines. To overcome this difficulty, we propose LR, Wald and Score tests for equaliy of C.Vs and Inverse Coefficient of Variations (ICV) from a Bivariate Normal Distribution. The finite sample performances of the tests are examined using

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extensive simulation. The simulation results indicate that the Wald test based on the ICV performs well and has more power compared to LR and Score tests using C.V and ICVs.

The organization of the paper is as follows.

In section 2, six tests are derived for equality of C.Vs from a Bivariate Normal Distribution using C.V and I.C.V. For small sample performance of the tests in terms of estimated type I error rate are examined in section 3. The paper concludes in section 4 where final remarks regarding the tests are provided.

2. Tests for Equality of C.Vs from a Bivariate Normal Distribution.

Let (x_1,y_1), ..., (x_n,y_n) be a random sample from a Bivariate Normal Distribution with pdf

$$f(x, y)=\frac{1}{\sigma_1\ \sigma_2\ 2\pi\left(1-\rho^2\right)^{1/2}}\ e-\frac{1}{2\left(1-p^2\right)}\left\{\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2+\left(\frac{y_i-\mu_2}{\sigma_2}\right)^2-\frac{2p}{\sigma_1\ \sigma_2}(x_i-\mu_1)(y_i-\mu_2)\right\}$$

$-\infty < x,\ y < \infty$, $-\infty < \mu_1,\ \mu_2 < \infty$ $\sigma_1, \sigma_2 > 0$, $-1 \le \rho \le 1$

In this paper we have used observed Fisher Information matrix for constructing Wald and Score tests. The reason is that Hinkley and Efron [14] advocate the use of observed Fisher Information rather than the expected Fisher Information. The score vector and the element from the Fisher Information matrix are given in the appendix. The hypothesis of interest is

H_0: $\eta_1=\eta_2$ where $\eta_1=\sigma_1/\mu_1$, $\eta_2=\sigma_2/\mu_2$

H_1: $\eta_1\neq\eta_2$

The Likelihood Ratio, Wald and Score test statistic for this hypothesis are given by

$\Lambda = -2\ln \lambda$,

$w = \frac{\hat{\eta}_1-\hat{\eta}_2}{\sqrt{V(\hat{\eta}_1-\hat{\eta}_2)}}$ Where $\hat{\eta}_1 = \frac{\hat{\sigma}_1}{\hat{\mu}_1}$, $\hat{\eta}_2 = \frac{\hat{\sigma}_2}{\hat{\mu}_2}$

And S= $U(\hat{\eta}_0)I(\hat{\eta}_0)^{-1}U(\hat{\eta}_0)'$

Testing for equality of C.Vs is equivalent for testing equality of ICVs. Sharma and Krishna [15] and Singh [13] advocate the use of inverse sample C.V (ISCV) for constructing Confidence Interval for the C.V of a distribution. The reason is that the Taylor series expansion for ISCV contains less number of terms than the sample C.V. Following this idea Nairy and Rao [16] developed LR, Wald and Score tests for testing equality of C.Vs from independent Normal Distributions. Their simulation results indicated that the Wald test based on ICV maintains type I error rate and has more power compared to the LR and Score tests. Inthis paper we derive the LR, Wald and Score tests for testing the hypothesis

H_0: $\theta_1 = \theta_2 = \theta$ (unknown), where $\theta_1 = \frac{\mu_1}{\sigma_1}$, $\theta_2 = \frac{\mu_2}{\sigma_2}$ H_1: $\theta_1\neq\theta_2$

The LRT statistic is given by $\Lambda = -2\ln \lambda$

The Wald test statistic is given by $w = \frac{\hat{\theta}_1-\hat{\theta}_2}{\sqrt{V(\hat{\theta}_1-\hat{\theta}_2)}}$ where $\hat{\theta}_1 = \frac{\hat{\mu}_1}{\hat{\sigma}_1}$, $\hat{\theta}_2 = \frac{\hat{\mu}_2}{\hat{\sigma}_2}$

The Score test statistic is given by S= $U(\hat{\theta}_0)\ I(\hat{\theta}_0)^{-1}U(\hat{\theta}_0)'$ where $I(\hat{\theta}_0) = -E\left(\frac{\partial^2 \log L(\hat{\theta}_0|x)}{\partial\theta\partial\theta'}\right)$.

3. Finite sample comparison of the Tests

3.1 Simulation Experiment

The asymptotic null distribution of the entire test statistic is central Chi-square with one degree of freedom and non-central Chi-square with the same no centrality parameter under the alternative hypothesis. For estimating size of the test samples of size n is generated from a Bivariate Normal Distribution with parameters μ_1, σ_1, μ_2, σ_2 and ρ. The values of μ_1, σ_1, μ_2, σ_2 are adjusted such that

$\sigma_1/\mu_1 = \sigma_2/\mu_2$ i.e. $\theta_1=\theta_2$ or inotherwords the C.V for the components of the Bivariate Normal Distribution are equal. The experiment is repeated 10,000 times and size of the test is proportion of the times the null hypothesis is rejected. Using the 10,000 simulated values of the test statistic the upper percentile value of the null distribution of the test statistic is recorded. The common values of $\theta_1=\theta_2=\theta$ are from 0.1 to 0.9 with an increment of 0.1. The values of the correlation coefficient ranges from -0.9 to 0.9 with an increment of 0.2. In addition to these values $\rho=0$ is also included. The samples are of sizes n=5, 10, 20, and 40. The value $\alpha=0.05$.

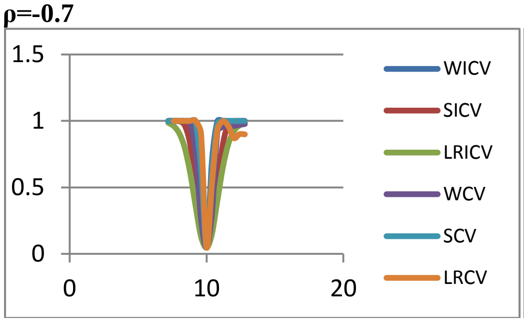
3.2 Estimated Type I Error

From the estimated type I error rates it follows that Wald test for I.C.V maintains type I error rate for all the sample sizes for all values of C.V and correlation coefficient. We say that a test maintains type I error rate if the size of the test is in the interval 0.045 ± 0.005. Likelihood Ratio and Score test for C.V and I.C.V. do not maintain type I error rate. The estimated error rates are far below the nominal level 0.05 and are the stringent tests. Score test for I.C.V. maintains type I error rate for all values of correlation coefficient and for $C.V \leq 0.4$.

3.3 Estimated Power Function

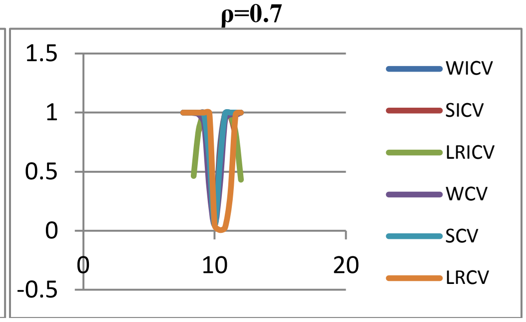
For comparing the power of various tests, it is important to ensure that all the tests have the same size. Since four of the tests do not maintain type I error rate for the computation of the power of the test, for Likelihood Ratio and Score tests using C.V and I.C.V the estimated α^{th} percentile values are used for estimating the power and for the Wald test the upper α percentile value of the Chi-square distribution with 1 degree of freedom is used. Fixing the value $\theta_1=\mu_1/\sigma_1=\theta$, the value used for estimating the type I error rates. θ_2 is adjusted such that $\theta_2=k\theta_1$. To ensure the value of θ_2, the standard deviation σ_2 is made equal to σ_1 and the values of μ_2 are altered. The power function for each test is estimated for fixed value of $\sigma_1=\sigma_2=\sigma$ and ρ. Figures 3.3(a) and (b) represent the power function for n=20, $\theta=0.1$, $\rho=-0.7$ and $\rho=0.7$

ρ=-0.7



3.3a

ρ=0.7



3.3b

3.4 Discussion

From the estimated Type I error rates and power of the test for all the configurations (9x11x4=396) it follows that Wald test for I.C.V maintain Type I error rate. When the power function of all the 6 tests are compared Wald test based on I.C.V emerges as the best test. The conclusion is based by comparing the power of the modest departures from the null hypothesis and rate of convergence of the power function to 1 in the right and left directions. Score test based on I.C.V emerges as the next best test. Wald test based on I.C.V has marginally higher power for modest alternatives compared to the Wald test based on C.V: The salient difference is that the rate of convergence of the power function to 1 is faster for the Wald test based on I.C.V compared to Wald test for the C.V.

The power function for the Likelihood Ratio test indicate that when the numerical value of coefficient of correlation is high (ignoring the sign), the power function of the Likelihood Ratio test exhibits irregular pattern, in the sense that the function increases in either direction and starts declining after a point in right and left directions. For the score test when the numerical value of the correlation coefficient is high, the power function of the test increases up to a level and then starts fluctuating, in the sense that it decreases and the increases and decreases. The oddities of the score test are examined in the paper of Sumathi and Rao [17]. For the Likelihood Ratio test a possible explanation for this is that the Likelihood is not well behaved when the coefficient of correlation is high, which creates difficulty in the estimation of restricted maximum likelihood estimators of the parameters.

Among Likelihood Ratio and Score tests, score test has higher power for modest alternative compared to Likelihood Ratio test. This is true for the tests based on C.V as well as I.C.V. Further the rate of convergence of the power function to 1 is faster for the

Score test than the Likelihood Ratio test. A salient finding is that all the tests based on I.C.V perform better than the tests based on C.V.

Sharma and Krishna [15] advocated the use of Inverse Sample C.V (ISCV) for construction of confidence interval for C.V. Further Singh [13] observed that Taylor series expansion for ISCV consists of fewer numbers of terms compared to sample C.V. Our results for tests based on C.V and I.C.V confirm their findings.

The power comparison of the tests for ρ=0 corresponds to the scenario when the tests based on correlated samples is used for testing the equality of C.V when the samples are independent. The present investigation reveals that one can safely use these proposed tests in the absence of a knowledge regarding the independence of the samples.

4. Conclusion:

In this paper we derived LR, Wald and Score tests based on C.V and I.C.V for testing equality of C.V of a Bivariate Normal Distribution. Testing for common value of C.V is also equivalent for testing for common value of I.C.V and thus LR, Wald and Score tests are derived for testing the equality of I.C.V. The tests are simple in nature. The finite sample comparison of simulation result indicates that Wald test based on I.C.V maintains type I error rate. LR and Score test needs a correction factor so that the accuracy of the Chi-square distribution and they maintain type I error rates. One alternative is to use bootstrap tests [18] and the other one is to use empirical satterthwaite approximation [19]. This re sampling technique may not appeal to the applied researchers. The power comparison for the various tests indicate that Wald test based on I.C.V has more power at modest alternatives compared to the other test and rate of convergence of the power function is faster for this test. Therefore we recommend this test for the users. The specific reason is the following.

1) This test maintains nominal level of significance. Whereas other tests require a correction for the test statistic. The correctionsbased on re sampling methods like Jackknife and bootstrap are tedious to implement.

2) Wald test does not require the estimation of restricted maximum likelihood estimator of the parameter, which cause problem when the samples are highly correlated.

3) The Wald test is more powerful for modest alternatives compared to the other tests. These alternatives are important from a practical perspective.

4) Computation of Wald test is simple compared to the other tests.

Mat lab program for carrying out the tests is available with the first author and can be made available to any interested person.

Appendix

$$H_o:\theta_1=\theta_2 \text{ Vs } \quad H_1:\theta_1\neq\theta_2 \qquad \theta_1=\theta_2=\theta$$

$$\log L = C-\frac{n}{2}\log\sigma_1^2-\frac{n}{2}\log\sigma_2^2-\frac{n}{2}\log\left(1-\rho^2\right)-\frac{1}{2\left(1-\rho^2\right)}\left\{\frac{\Sigma x_i^2}{\sigma_1^2}+\frac{\Sigma y_i^z}{\sigma_2^2}+\right.$$
$$\left.2n\theta^2-2\theta\left(\frac{\Sigma x_i}{\sigma_1}+\frac{\Sigma y_i}{\sigma_2}\right)\right\}+\left(\frac{\rho}{1-\rho^2}\right)\left\{\frac{\Sigma x_i y_i}{\sigma_1\,\sigma_2}-\theta\left(\frac{\Sigma x_i}{\sigma_1}+\frac{\Sigma y_i}{\sigma_2}\right)+n\theta^2\right\}$$

$$\frac{\partial\log L}{\partial\theta}=\frac{1}{1+\rho}\left\{\frac{\Sigma x_i}{\sigma_1}+\frac{\Sigma y_i}{\sigma_2}-2n\theta\right\},\quad \frac{\partial^2\log L}{(\partial\theta)^2}=\frac{-2n}{1+\rho},\quad \frac{\partial^2\log L}{\partial\theta\,\partial\sigma_2^2}=\frac{-\Sigma y_i}{2\sigma_2^3(1+\rho)}=\frac{\partial^2\log_L}{\partial\sigma_2^2\,\partial\theta}$$

$$\frac{\partial^2\log L}{\partial\theta\,\partial\sigma_1^2}=\frac{-\Sigma x_i}{2\sigma_1^3(1+\rho)}=\frac{\partial^2\log_L}{\partial\sigma_1^2\partial\theta},\qquad \frac{\partial^2\log L}{\partial\theta\,\partial\rho}=\frac{-1}{(1+\rho)^2}\left\{\frac{\Sigma x_i}{\sigma_1}+\frac{\Sigma y_i}{\sigma_2}-2n\theta\right\}=\frac{\partial^2\log_L}{\partial\rho\,\partial\theta}$$

$$\frac{\partial\log L}{\partial\sigma_1^2}=\frac{-n}{2\sigma_1^2}+\frac{1}{\sigma_1^3\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i^2}{\sigma_1}-\Sigma x_i\theta(1+\rho)-\frac{\rho\Sigma x_i y_i}{\sigma_2}\right\}$$

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$$\frac{\partial^2 \log L}{\left(\partial\sigma_1^2\right)^2} = \frac{n}{2\left(\sigma_1^2\right)^2} - \frac{1}{2\sigma_1^5\left(1-\rho^2\right)}\left\{\frac{2\Sigma x_i^2}{\sigma_1} - \frac{3\theta\,\Sigma x_i}{2}\right\} + \frac{3\rho}{4\sigma_1^5\left(1-\rho^2\right)}\left\{\frac{\Sigma x_i y_i}{\sigma_2} - \theta\,\Sigma x_i\right\} \quad \frac{\partial^2 \log L}{\partial\sigma_1^2\partial\sigma_2^2} = \frac{\rho\Sigma x_i y_i}{4\sigma_1^3\sigma_2^3\left(1-\rho^2\right)} = \frac{\partial^2 \log_L}{\partial\sigma_2^2\partial\sigma_1^2}$$

$$\frac{\partial^2 \log L}{\partial\sigma_1^2\partial\rho} = \frac{\rho}{\left(1-\rho^2\right)^2\sigma_1{}^3}\left\{\frac{\Sigma x_i}{\sigma_1} - \Sigma x_i\right\} - \left\{\frac{1+\rho^2}{2\left(1-\rho^2\right)^2\sigma_1^3}\right\}\left\{\frac{\Sigma x_i y_i}{\sigma_2} - \theta\right\} = \frac{\partial^2 \log_L}{\partial\rho\partial\sigma_1^2}$$

$$\frac{\partial \log L}{\partial\sigma_2^2} = \frac{-n}{2\sigma_2^2} + \frac{1}{\sigma_2^3\left(1-\rho^2\right)^2}\left\{\frac{\Sigma y_i^2}{\sigma_2} - \theta(1+\rho)\,\Sigma y_i - \frac{\rho\,\Sigma x_i y_i}{\sigma_1}\right\}$$

$$\frac{\partial^2 \log L}{\left(\partial\sigma_2^2\right)^2} = \frac{n}{2\left(\sigma_2^2\right)^2} - \frac{1}{2\left(1-\rho^2\right)\sigma_2^5}\left\{\frac{2\,\Sigma y_i^2}{\sigma_2} - \frac{\theta_3\,\Sigma y_i}{2}\right\} + \frac{3\rho}{4\sigma_2^5\left(1-\rho^2\right)}\left\{\frac{\Sigma x_i y_i}{\sigma_1} - \theta\,\Sigma y_i\right\}$$

$$\frac{\partial^2 \log L}{\partial\sigma_2^2\partial\rho} = \frac{\rho}{\left(1-\rho^2\right)^2\sigma_2^3}\left\{\frac{\Sigma y_i}{\sigma_2} - \theta\,\Sigma y_i\right\} - \left(\frac{1+\rho^2}{2\sigma_2^3\left(1-\rho^2\right)^2}\right)\left\{\frac{\Sigma x_i y_i}{\sigma_1} - \theta\,\Sigma y_i\right\} = \frac{\partial^2 \log_L}{\partial\rho\;\partial\sigma_2^2}$$

$$\frac{\partial^2 \log L}{\partial\rho} = \frac{n\rho}{1-\rho^2} - \frac{\rho}{\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i^2}{\sigma_1^2} + \frac{\Sigma y_i^2}{\sigma_2^2} + 2n\theta^2 - 2\theta\left(\frac{\Sigma x_i}{\sigma_1} + \frac{\Sigma y_i}{\sigma_2}\right)\right\} + \frac{1+\rho^2}{\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i y_i}{\sigma_1\,\sigma_2} - \theta\left(\frac{\Sigma x_i}{\sigma_1} + \frac{\Sigma y_i}{\sigma_2}\right) + n\theta^2\right\}$$

$$\frac{\partial \log L}{\partial\theta_1} = \frac{-1}{1-\rho^2}\left\{n\theta_1 - \rho n\theta_2 - \frac{\Sigma x_i}{\sigma_1} + \frac{\rho\,\Sigma y_i}{\sigma_2}\right\}, \quad \frac{\partial^2 \log L}{\partial\theta_1^2} = \frac{-n}{1-\rho^2} \quad \frac{\partial^2 \log L}{\partial\theta_1\;\partial\theta_2} = \frac{n\rho}{\left(1-\rho^2\right)} = \frac{\partial^2 \log^2}{\partial\theta_2\;\partial\theta_1},$$

$$\frac{\partial^2 \log L}{\partial\theta_1\;\partial\sigma_1^2} = \frac{-\Sigma x_i}{2\sigma_1^3\left(1-\rho^2\right)} = \frac{\sigma^2 \log_L}{\partial\sigma_1^2\;\partial\theta_1} \quad \frac{\partial^2 \log L}{\partial\theta_1\;\partial\sigma_2^2} = \frac{\rho\,\Sigma y_i}{2\sigma_2^3\left(1-\rho^2\right)} = \frac{\sigma^2 \log_L}{\partial\sigma_2^2\;\partial\theta_1},$$

$$\frac{\partial^2 \log L}{\partial\theta_1\;\partial\rho} = \frac{1}{\left(1-\rho^2\right)^2}\left\{2\rho\left(\frac{\Sigma x_i}{\sigma_1} - n\theta_1\right) + \left(1+\rho^2\right)\left(n\theta_2 - \frac{\Sigma y_i}{\sigma_2}\right)\right\} \quad \frac{\partial l \log L}{\partial\theta_2} = \frac{1}{1-\rho^2}\left\{n\theta_2 - \frac{\Sigma\,y_i}{\sigma^2} + \frac{\rho\,\Sigma x_i}{\sigma_1} - \rho\,n\theta_1\right\},$$

$$\frac{\partial^2 \log L}{(\partial\theta_2)^2} = \frac{-n}{1-\rho^2} \quad \frac{\partial^2 \log L}{\partial\theta_2\;\partial\sigma_1^2} = \frac{\rho\,\Sigma x_i}{2\sigma_1^3\left(1-\rho^2\right)} = \frac{\partial^2 \log_L}{\partial\sigma_1^2\;\partial\theta_2}, \qquad \frac{\partial^2 \log L}{\partial\theta_2\;\partial\theta_1} = \frac{n\rho}{1-\rho^2} = \frac{\partial^2 \log_L}{\partial\theta_1\;\partial\theta_2}$$

$$\frac{\partial^2 \log L}{\partial\theta_2\;\partial\sigma_2^2} = \frac{-\Sigma y_i}{2\sigma_2^3\left(1-\rho^2\right)} = \frac{\partial^2 \log^2}{\partial\sigma_2^2\;\partial\theta_2}, \frac{\partial^2 \log L}{\partial\theta_2\partial\rho} = \frac{1}{\left(1-\rho^2\right)^2}\left\{2\rho\left(\frac{\Sigma y_i}{\sigma_2} - n\theta_2\right) - \left(1+\rho^2\right)\left(\frac{\Sigma x_i}{\sigma_1} - n\theta_1\right)\right\}$$

$$\frac{\partial \log L}{\partial\sigma_1^2} = \frac{-n}{2\sigma_1^2} - \frac{1}{2\left(1-\rho^2\right)}\left\{\frac{\Sigma x_i}{\sigma_1^3}(\theta_1 + \rho\theta_2) - \frac{\Sigma x_i^2}{\left(\sigma_1^2\right)^2} + \frac{\rho\,\Sigma x_i\,y_i}{\sigma_1^3\;\sigma_2}\right\}$$

$$\frac{\partial^2 \log L}{\left(\partial\,\sigma_1^2\right)^2} = \frac{n}{\left(2\sigma_1^2\right)^2} - \frac{1}{2\left(1-\rho^2\right)}\left\{\frac{2\Sigma x_i^2}{\left(\sigma_1^2\right)^3} - \frac{3\Sigma x_i\,(\theta_1 + \rho\theta_2)}{\sigma_1^5} - \frac{3\rho\,\Sigma x_i y_i}{4\sigma_2\sigma_1^5}\right\} \quad \frac{\partial^2 \log L}{\partial\sigma_1^2\partial\sigma_2^2} = \frac{\rho\,\Sigma x_i y_i}{4\left(1-\rho^2\right)\sigma_1^3\;\sigma_2^3} = \frac{\partial^2 \log_L}{\partial\sigma_2^2\;\partial\sigma_1^2}$$

$$\frac{\partial^2 \log L}{\partial\sigma_1^2\;\partial\rho} = \frac{-\rho}{\left(1-\rho^2\right)^2}\left\{\frac{\theta_1\,\Sigma x_i}{\sigma_1^3} - \frac{\Sigma x_1^2}{\left(\sigma_1^2\right)^2}\right\} - \left\{\frac{\theta_2\,\Sigma x_i}{\sigma_1^3} + \frac{\Sigma x_i y_i}{\sigma_1^3\sigma_2}\right\}\left\{\frac{1+\rho^2}{2\left(1-\rho^2\right)^2}\right\}$$

$$\frac{\partial \log L}{\partial\sigma_2^2} = \frac{-n}{2\sigma_2^2} - \frac{1}{2\left(1-\rho^2\right)}\left\{\frac{\Sigma y_i}{\sigma_2^3}(\theta_2 + \rho\theta_1) - \frac{\Sigma y_i^2}{\left(\sigma_2^2\right)^2} + \frac{\rho\Sigma x_i y_i}{\sigma_1\;\;\sigma_2^3}\right\}$$

$$\frac{\partial^2 \log L}{\left(\partial\sigma_2^2\right)^2} = \frac{n}{2\left(\sigma_2^2\right)^2} - \frac{1}{2\left(1-\rho^2\right)}\left\{\frac{2\Sigma y_i^2}{\left(\sigma_2^2\right)^3} - 3(\theta_2 + \rho\theta_1)\frac{\Sigma y_i}{\sigma_2^5} - \frac{3\rho\Sigma x_i y_i}{4\sigma_1\sigma_2^5}\right\}$$

$$\frac{\partial^2 \log L}{\partial\sigma_2^2\partial\rho} = \frac{\rho}{\left(1-\rho^2\right)^2}\left\{\frac{\Sigma y_i^2}{\left(\sigma_2^2\right)^2} - \frac{\theta_2\Sigma y_i}{\sigma_2^3}\right\} - \frac{1+\rho^2}{2\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i y_i}{\sigma_1\sigma_2^3} + \frac{\theta_1\Sigma y_i}{\sigma_2^3}\right\}$$

$$\frac{\partial \log L}{\partial\rho} = \frac{-\rho}{\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i^2}{\left(\sigma_1^2\right)} + \frac{\Sigma y_i^2}{\sigma_2^2} + n\left(\theta_1^2 + \theta_2^2\right) - \frac{2\theta_1\Sigma x_i}{\sigma_1} - \frac{2\theta_2\Sigma y_i}{\sigma_2}\right\} + \frac{1+\rho^2}{\left(1-\rho^2\right)^2}\left\{\frac{\Sigma x_i y_i}{\sigma_1\sigma_2} - \frac{\theta_2\Sigma x_i}{\sigma_1} - \frac{\theta_1\Sigma y_i}{\sigma_2} + n\theta_1\theta_2\right\} + \left(\frac{n\rho}{1-\rho^2}\right)$$

$$\frac{\partial^2 \log L}{(\partial\rho)^2} = -\left[\frac{1+3\rho^2}{\left(1-\rho^2\right)^3}\right]\left\{\frac{\Sigma x_i^2}{\sigma_1^2} + \frac{\Sigma y_i^2}{\sigma_2^2} + n\left(\theta_1^2 + \theta_2^2\right) - \frac{2\theta_1\Sigma x_i}{\sigma_1} - \frac{2\theta_2\;\Sigma y_i}{\sigma_2}\right\} + \frac{2\rho\left(3+\rho^2\right)}{\left(1-\rho^2\right)^3}\left\{\frac{\Sigma x_i y_i}{\sigma_1\;\sigma_2} - \frac{\theta_2\Sigma x_i}{\sigma_1} - \frac{\theta_1\Sigma y_i}{\sigma_2} + n\theta_1\;\theta_2\right\} + \frac{n\left(1+\rho^2\right)}{\left(1-\rho^2\right)^2}$$

References

1. Miller E. G. and Karson (1977) *Testing equality of two coefficients of variation*, Am. Stat. Assoc. proc. Bus. Econ. Sect. Part I, 278-283
2. McKay, A.T.(1932), *Distribution of the Co-efficient of variation and the extended 't' distribution*. J.Royal.Stat.Soc.**95,** *696-698.*
3. Shipra Banik and Kibra Golam B. M. (2011), *Estimating the Population Coefficient of Variation by Confidence Intervals,* Commn. In Stat. Simul.Comp, 40, Issue 8.
4. Bennett B. M. (1977), *LR Tests For Homogeneity of Coefficients of Variation inRepeated Samples*, Snaky, 39, 400-405
5. Shafer, N. J. and J. A. Sullivan. (1986), *A Simulation study of a test for the equality of the Coefficients of Variation.* Commn. Stat. Computat, 15, 681-695.
6. Doornbos R. and Dijkstra, J.B. (1983), *A Multi sample Test for the equality ofcoefficients of variation in Normal populations*, commn. In stat-sim. And comp. 12(21, 147-158)
7. Rao, K.A. and Bhat, A.R.S. (1989): A note on test for coefficient of variation. *Calcutta Statistical Association Bulletin*, 38, 225-229.
8. Miller E.G (1991), *Asymptotic Test Statistics for Coefficient of Variation,* Commn. In Stats. Th. and Methods, 20(10), 3351-3363.
9. Gupta and Ma (1996), *Testing the equality of Coefficients of Variation in k normal population,* Commn. In Stats. Th. and Methods, vol.25, Issue 1198.
10. Weerahandi Samaradasa and Zidek Zim (1989), *Generalized p-values in Significance Testing of Hypothesis in the Presence of Nuisance Parameters,* Jourl. Of the American Statl. Assoc. ,vol.4, Issue 406.
11. Jafari A. A. and Behboodian J (2010), *Two Approaches for Comparing the Coefficientsof Variation of Several Normal Populations*. , World Appl. Sci. J. , 10(7): 853-857
12. Cabras, Mostallino and Racugno (2006) *A nonparametric bootstrap test for the equalityof coefficients of variation.,* Commu. In stat.-simu. And compu., vol. 35, Issue 3, 2006.
13. Murari Singh (1993), *Behavior of Sample Coefficient of Variation drawn from Several distributions*., Sankya, 55, series B., Pt 1, 65-76
14. Bradely Efron and David Hinkley (1978), *Assessing the accuracy of the maximum likelihood estimator: Observed verses expected Fisher information.,* Biometrica 65(3); 457-483.
15. Sharma and Krishna (1994) *Asymptotic sampling distribution of inverse coefficient of variation and its applications*., IEEE Trans. Reliab. 43(4), 630-633
16. Nairy, K.S., Rao, K. A. (2003), *Tests of Coefficients of Variation of Normal Population.,* Commn. In Stat. Simul. Comp. 32, 3, 641-646
17. Sumathi K. and Rao A. (2011), *A Note on Inconsistency of the Score Test.,* Pakistani Jourl. of Stats and Operation Research, vol. 7, No1.
18. Erfon B. and Tibshirani R.J. (1993), *An Introduction to the Bootstrap*, Chapman and Hall/CRC Monographs on statistics and Applied Probability.
19. Vasudeva Guddattu and Aruna Rao K. (2011), *Modified Satterwaite bootstrap tests for Inflate parameter in zero inflated Negative binomial distribution.* 2nd IJMA International Conference on Advanced Data Analysis, Business Analytics and Intelligence.

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