

Performance of Maximum ratio combining (MRC) MIMO Systems for Rayleigh Fading Channel

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Abstract: The effect of fading and interference effects can be combated with equalizer for a MIMO system. MIMO systems exploit the multipath propagation in rich scattering environment using multiple transmit and receive antennas to increase the capacity of a link. In this contribution, we provide an exact BER analysis for fading channels with maximal-ratio combining (MRC) and imperfect channel estimation at the receive. The BER characteristics for the various transmitting and receiving antennas simulated in MATLAB tool box and it shows that the MRC equalizer based receiver is a good choice for removing some ISI and minimizes the total noise power. The results show that the BER decreases as the $m \times n$ antenna configurations is increased.

Index term- Bit Error Rate, EGT, MIMO, Multipath fading

I. INTRODUCTION

The performance of wireless communication systems is mainly governed by the wireless channel environment. As opposed to the typically static and predictable characteristics of a wired channel, the wireless channel is rather dynamic and unpredictable, which makes an exact analysis of the wireless communication system often difficult. In recent years, optimization of the wireless communication system has become critical with the rapid growth of mobile communication services. In wireless communication, radio propagation refers to the behavior of radio waves when they are propagated from transmitter to receiver. In the course of propagation, radio waves are mainly affected by three different modes of physical phenomena: reflection, diffraction, and scattering. A unique characteristic in a wireless channel is a phenomenon called 'fading,' the variation of the signal amplitude over time and frequency. In contrast with the additive noise as the most common source of signal degradation, fading is another source of signal degradation that is characterized as a non-additive signal disturbance in the wireless channel. Fading may either be due to multipath propagation, referred to as multi-path (induced) fading, or to shadowing from obstacles that affect the propagation of a radio wave, referred to as shadow fading. The fading phenomenon in the wireless communication channel was initially modeled for HF, UHF, SHF bands. Currently, the most popular wireless channel models have been established for 800MHz to 2.5 GHz by extensive channel measurements in the field. Includes the ITU-R standard channel models specialized for SISO. For a multi-antenna communication system, referred to as the MIMO (Multiple Input Multiple Output) system, have been recently developed by the

various research and standardization activities, aiming at high-speed wireless transmission and diversity gain[1]. Diversity combining is a well-known technique to mitigate the performance degradation of multipath fading and co channel interference (CCI) in wireless systems. In flat fading channels, maximal ratio combining (MRC) diversity is well known to be optimum in the sense of maximizing the output signal-to-noise ratio (SNR). If the desired signal s affected by both co-channel interference (CCI) and flat fading, the diversity combining technique that maximizes the output signal-to-interference-plus-noise ratio (SINR) is the so called optimum combining (OC). However, OC is much more complex than MRC and typically requires information about the CCI that may not be available at the receiver. Thus, in practice many wireless systems will use MRC even in the presence of CCI. The transmit antenna array can also be used to provide diversity gain, and the optimum technique under background noise is Maximal Ratio Transmission (MRT), equivalent to MRC. MIMO systems employing both MRT and MRC are usually referred to as MIMO MRC [2]. Various transmit diversity techniques have been proposed in the open literature, delay transmit diversity scheme [3] and transmit diversity is a simple but effective scheme proposed by Alamouti [4]. However, these transmit diversity techniques were built on objectives other than to maximize the SNR. That is, they are suboptimum in terms of SNR performance. Accordingly, the frame work of maximum ratio Combining (MRC) will be established here in terms of concept and principles[5]. It is well known that maximal-ratio combining (MRC) [6] is the optimal linear combining technique. However, with receiver MRC, most of the system complexity concentrates at the receiver side. To decrease the receiver complexity in terms of the number of RF chains, a simple suboptimal combining scheme, referred to as selection combining (SC), was proposed in [7], in which only one receive antenna with the largest signal-to-noise ratio (SNR) is selected for demodulation. The SC scheme has been extended to the cases where the signals on more than one receive antenna with the largest instantaneous SNRs are combined [8]. This scheme is referred to as maximal-ratio combining. MRC is a powerful technique. It is most common in SIMO channels. This paper presents the concept, principles, and analysis of maximum ratio transmission for wireless communications, where multiple antennas are used for both transmission and reception. The principles and analysis are applicable to general cases, including maximum-ratio combining. Simulation results agree with the analysis. Analysis includes simulated result for the no of receiving channel $V_s E_b/N_0$, BER of different modulation schemes for MRT-MRC, providing a performance comparison of systems

II. SYSTEM MODEL

Various techniques are known to combine the signals from multiple diversity branches. In Maximum Ratio combining each signal branch is multiplied by a weight factor that is proportional to the signal amplitude. That is, branches with strong signal are further amplified, while weak signals are attenuated. In telecommunications, maximal-ratio combining is a method of diversity combining in which the signals from each channel are added together and the gain of each channel is made proportional to the RMS value of signal and inversely proportional to the mean square noise level in that channel. Different proportionality constants are used for each channel. It is also known as ratio-squared combining and predetection combining. Maximal-ratio-combining is the optimum combiner for independent AWGN channels[9]. In maximal ratio combining (MRC), the signals from all of the MR branches are weighted according to their individual SNRs and then summed. Here the individual signals need to be brought into phase alignment before summing. This implies

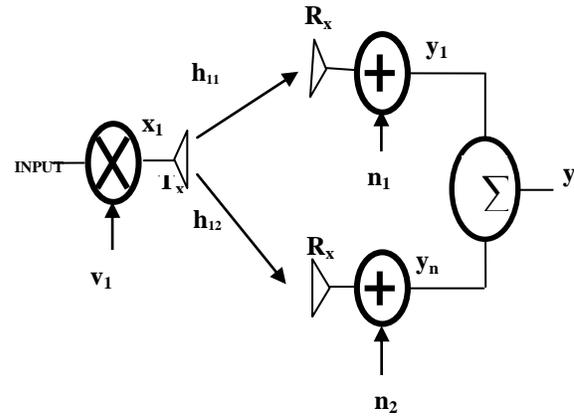


Fig.2 Maximum ratio combining with 1 Tx and 2 Rx.

Fig .2 indicates Maximum ratio combining with 1 Tx and 2 Rx representation. The symbol to be transmitted is weighted with a transmit weighting vector to form the transmitted signal vector. The received signal vector is the product of the Transmitted signal vector and the channel plus the noise; that is

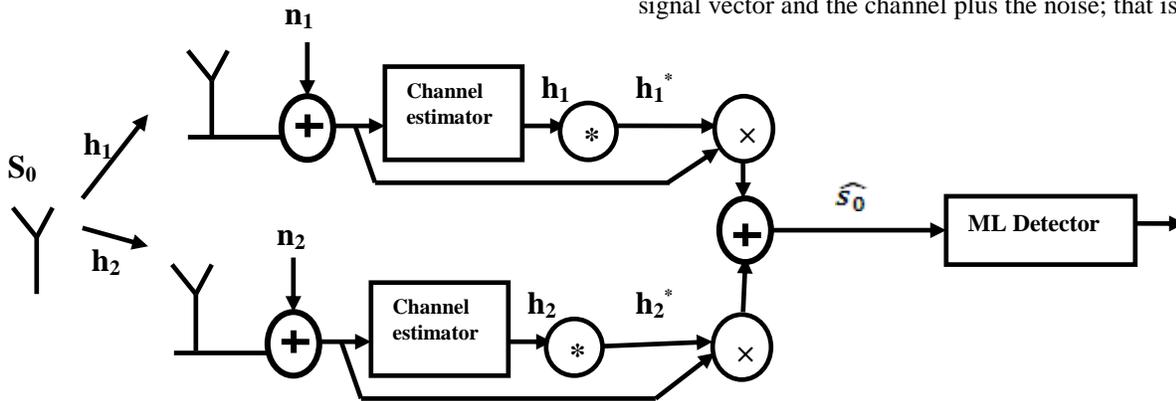


Fig.1 Block diagram of MRC

Analysis individual RF receiver tracts. In the case of maximum ratio combining shown in fig 1, the resulting received signals are

$$r_1 = h_1 s_0 + n_1 \dots \dots \dots (1)$$

$$r_2 = h_2 s_0 + n_2 \dots \dots \dots (2)$$

and the combined signal is

$$\tilde{s}_0 = h_1 * r_1 + h_2 * r_2 \dots \dots \dots (3)$$

Where the fading coefficients from antennas 1 and 2 are defined by h1 and h2, respectively, at time t. n1 and n2 are independent complex variables with zero mean and unit variance, representing additive white Gaussian noise samples at time t. [10].

$$Y = H * x + n \dots \dots \dots (4)$$

Where the transmitted signals x is given by

$$X = [X_1 \dots \dots \dots X_K]^T \dots \dots \dots (5)$$

$$= C[V_1 \dots \dots \dots V_K]^T \dots \dots \dots (6)$$

a system is considered, which consists of K antennas for transmission and L antennas for reception. The channel consists of K* L statistically independent coefficients, as shown in Fig. 2. It can be conveniently represented by a matrix

$$H = \begin{bmatrix} H_{11} & \dots & H_{1K} \\ \vdots & \ddots & \vdots \\ H_{L1} & \dots & H_{LK} \end{bmatrix} \dots \dots \dots (7)$$

The entries of \mathbf{H} are modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and variance 0.5 per dimension and the elements of \mathbf{n} are modeled as i.i.d. complex Gaussian random variables with zero mean and variance $N_0/2$ per dimension. Rayleigh distribution is the most representative of Non-Line of Sight (N-LOS) wireless radio propagation and hence the MIMO channel capacity has been investigated for Rayleigh fading channel model includes the concept of MRC. It is assumed that the channel state information (CSI) is known exactly at the receiver and not at the transmitter, and the channel fading coefficients remain constant over the entire frame and changes from one frame to another. The information-theoretic capacity of such full complex MIMO systems which use all available transmit and receive antennas is given by

$$C_{\text{full}} = \log_2 \det \left(I_{N_R} + \frac{E_s}{(N_T N_0)} \right) \quad \text{if } N_R < N_T$$

Where HH is component wise transpose conjugate of H . I_{N_R} is $N_R * N_R$ identity matrix

The noise vector is expressed as

$$\mathbf{n} = [\mathbf{n}_1 \dots \dots \dots \mathbf{n}_L]^T \dots \dots \dots (8)$$

Noise is assumed to be white Gaussian and uncorrelated with the signals. The received signals are weighted and summed to produce the estimate of the symbol.[11]-[16].As the mobile radio channel is a time-varying multipath channel and is subject to physical propagation path loss .The time-variations are caused by the medium changes as the vehicles moves. The propagation losses are related to both the atmospheric propagation and the terrain configuration. In [17], Sklar viewed that the multipath aspect is caused by different scatterers and reflectors such as building or trees that surround the mobile unit. As a result of these propagation phenomena in a narrow-band transmission, where narrow-band is defined with respect to the coherence bandwidth of the channel, the receive signal affect the performance of the receiver which results in an increase of bit error rate (BER). The channel performance is obtained for different modulation technique which viewed the four major performance categories in terms of signal to noise ratio (E_b/N_0) versus error probability [18]. we know that, if \mathbf{h}_i is a Rayleigh distributed random variable, then \mathbf{h}_i^2 is a chi-squared random variable with two degrees of freedom

The pdf of \mathcal{Y}_i is

$$p(\mathcal{Y}_i) = \frac{1}{(E_b/N_0)} e^{-\frac{\mathcal{Y}_i}{(E_b/N_0)}} \dots \dots \dots (11)$$

Since the effective bit energy to noise ratio \mathcal{Y} is the sum of N such random variables, the pdf of \mathcal{Y} is a chi-square random variable with $2N$ degrees of freedom. The pdf of \mathcal{Y} is,

$$p(\mathcal{Y}) = \frac{1}{(N-1)! (E_b/N_0)^N} \mathcal{Y}^{N-1} e^{-\frac{\mathcal{Y}}{(E_b/N_0)}}, \mathcal{Y} \geq 0 \dots (12)$$

If you recall, in the post on BER computation in AWGN, with bit energy to noise ratio of $\frac{E_b}{N_0}$, the bit error rate for BPSK in AWGN is derived as

$$P_b = \frac{1}{2} \text{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right) \dots \dots \dots (13)$$

Given that the effective bit energy to noise ratio with maximal ratio combining is \mathcal{Y} , the total bit error rate is the integral of the conditional BER integrated over all possible values of \mathcal{Y} [18].

This equation reduces to

$$P_e = p^N \sum_{k=0}^{N-1} (N-1+k)(1-p)^k \dots \dots \dots (14)$$

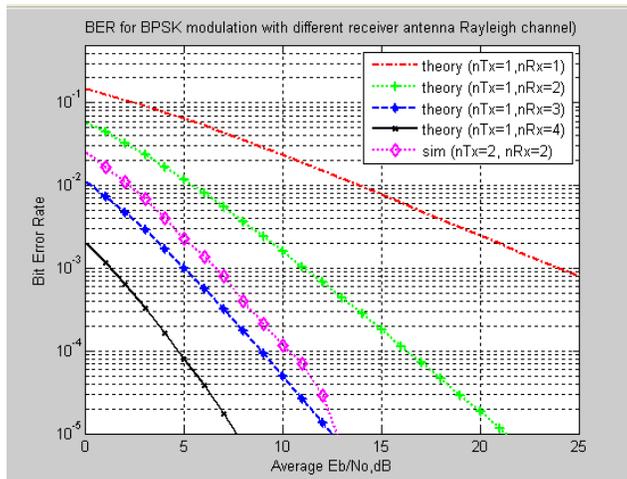
Where

$$p = \frac{1}{2} - \frac{1}{2} \left(1 + \frac{1}{E_b/N_0} \right)^{-1/2}$$

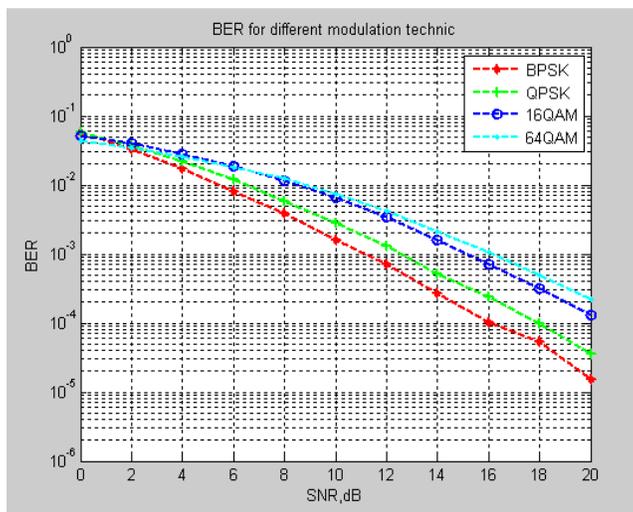
In uncoded systems under Rayleigh fading, the central limit theorem can be applied to the total interference, which can therefore be approximated to be Gaussian distributed. The average error rates can thus be calculated by remediating the conditional error probability (CEP), i.e., the error rate under AWGN, over the output SINR.

III. CONCLUSION

We have studied the improvement in the BER for modulation scheme BPSK, QPSK, 16- and 64-QAM system under MRC. The performance of the maximal ratio combiner that maximizes the output SNR in the presence of co channel interference and noise is analyzed. Analysis includes simulated result for the no of receiving channel Vs BER, BER of different modulation schemes for MRT-MRC, providing a performance comparison of systems.



From above graph we summarize that increasing number of receiving antenna, the BER decreases.



From this simulated result, by observing the graph resolution we conclude that if SNR value is less than or equal to 2 then we can choose 64 QAM modulation technique. But if we move further exceeding SNR Value greater than 2, we have the choice .BPSK Modulation is preferred over others as it have less BER value as compared to other modulation technique.

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