FUZZY SET GO- SUPER CONNECTED MAPPINGS

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Abstract- The aim of this paper is to introduce and discuss the concepts of fuzzy GO-super connectedness between fuzzy sets and fuzzy set GO–super connected mappings in fuzzy topological spaces.

Index Terms- Fuzzy topology, fuzzy continuity, fuzzy super interior, fuzzy super closure fuzzy supper closed set, fuzzy supper open set. fuzzy g- super closed sets, fuzzy g- super open sets, fuzzy super continuity ,fuzzy super GO-connectedness between fuzzy sets and fuzzy set super GO-connected mappings.

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I. PRELIMINARIES

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y)=\beta$ for y = x and $x_\beta(y) = 0$ for $y \ne x$,$\beta \in [0,1]$ and $y\in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x)>1$. A fuzzy set A is quasi–coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x)>1$.For any two fuzzy sets A and B of X, $\rceil(AqB) \Leftrightarrow A \le 1- B$.

Definition 1.1[7,8,12] Let $(X, \Im)$ be a fuzzy topological space and A be a fuzzy set in X. Then the fuzzy interior and fuzzy closure , fuzzy supeinterior and fuzzy supe closure of A are defined as follows;

$cl(A) = \cap\{K : K$ is an fuzzy closed set in X and $A \le K\}$,

$int(A) = \cup\{G : G$ is an fuzzy open set in X and $G \le A\}$.

Defination 2.1[7,8]: Let (X, Γ) be a fuzzy topological space then and a fuzzy set $A\subseteq X$ then

1. fuzzy supper closure and the of A by $scl(A) = \{x \in X : cl(U) \cap A \ne \phi\}$ for every fuzzy open set U containing x}
2. fuzzy supper-interior $sint(A) = \{x \in A : cl(U) \subseteq A$ for some fuzzy open set U containing x}, respectively.

Definition 2.2[7,8]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. Fuzzy super closed if $scl(A) \le A$.
2. Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Definition 1.2[7,8,9,14]: A fuzzy set A of an fuzzy topological space $(X,\Im)$ is called:

1. Fuzzy super closed if $scl(A) \le A$.
2. Fuzzy super open if 1-A is fuzzy super closed sint(A)=A
3. fuzzy g-closed if $cl(A) \le O$ whenever $A \le O$ and O is fuzzy open
4. fuzzy g-open if its complement 1- A is fuzzy g-closed.
5. fuzzy g-super closed if $cl(A) \le O$ whenever $A \le O$ and O is fuzzy upper open.
6. fuzzy g- super open if its complement 1- A is fuzzy g- super closed.

Remark 1.1[7,8,9,15]: Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. Fuzzy g- super open) but the converse may not be true.

Definition 1.3[7,8,9,12]: Let (X,τ) be a fuzzy topological space and A be a fuzzy set in X. Then the g-super interior and the g- super closure of A are defined as follows;

(a) $gscl(A) = \cap \{K : K$ is a fuzzy g- super closed set in X and $A \le K.\}$
(b) $gsint(A) = \cup \{G : G$ is a fuzzy g- super open set in X and $G \le A.\}$.

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Definition 1.4 [9]: A fuzzy topological space (X,τ) is said to be fuzzy super connected (resp. fuzzy super GO-connected) if no non-empty fuzzy set which is both fuzzy super open and fuzzy super closed (resp. fuzzy g- super open and fuzzy g- super closed).

Definition 1.5 [7,8,9]: A fuzzy topological space $(X, \Im)$ is said to be connected between fuzzy sets A and B if there is no fuzzy super closed super open set F in X such that $A \leq F$ and $\rceil(F_q B)$.

Definition 1.6[7,8,9,11]: A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy set super connected provided that if X is fuzzy super connected between fuzzy sets A and B , f(X) is fuzzy super connected between f(A) and f(B) with respect to relative fuzzy topology.

Remark1.2 [7,8,9,11]: Every fuzzy super continuous function is fuzzy set super connected but the converse may not be true.

Definition1.7 [1,7,8,9]: Let (X,τ) and (Y,ϕ) be two fuzzy topological space let f: X→be a function. Then f is said to be:

a. fuzzy g- super continuous if inverse image of every fuzzy closed set of Y is fuzzy g- super closed in X.
b. fuzzy weakly g- super continuous if $f^{-1}(B) \leq gsint(f^{-1}(cl(B))$ for each fuzzy super open set B of Y.
c. fuzzy almost g- super continuous if $f^{-1}(B) \leq gsint(f^{-1}(int(cl(B))))$ for each fuzzy super open set B of Y.

II. FUZZY GO-SUPER CONNECTEDNESS BETWEEN FUZZY SETS

Definition 2.1: A fuzzy topological space $(X, \Im)$ is said to be fuzzy super GO-connected between fuzzy sets A and B if there is no fuzzy g- super closed fuzzy g- super open set F in X such that $A \leq F$ and $\rceil(F_q B)$.

Theorem 2.1: If a fuzzy topological space $(X, \Im)$ is fuzzy super GO-connected between fuzzy sets A and B then it is fuzzy super connected between A and B.

Prof : If $(X, \Im)$ is not fuzzy super connected between A and B then $\exists$ a fuzzy super closed fuzzy super open set F in X such that $A \leq F$ and $\rceil(F_q B)$. Then by Remark1.1, F is a fuzzy g- super closed fuzzy g- super open set in X such that $A \leq F$ and $\rceil(F_q B)$. Hence $(X, \Im)$ is not fuzzy super GO-connected between A and B, which contradicts our hypothesis.

Remark 2.1: Converse of Theorem 2.1 may be false .For,

Example 2.1 Let X = { a ,b } and A and B be fuzzy sets in X defined as follow
U (a)=0.5, U(b) =0.6 , A(a) =0.2 , A(b)=0.3 , B(a) = 0.5 , B(b) = 0.4.
Let $\Im = \{\mathbf{0}, U, 1\}$ be fuzzy a fuzzy topology on X .Then $(X, \Im)$ is fuzzy super connected between A and B but it is not fuzzy super GO connected between A and B.

Theorem 2.2: A fuzzy topological space $(X, \Im)$ is fuzzy super GO-connected between fuzzy sets A and B if and only if there is no fuzzy g- super closed fuzzy g- super open set F in X such that $A \leq F \leq 1- B$.

Proof: Necessity: Let $(X, \Im)$ is fuzzy super GO-connected between fuzzy sets A and B. Suppose contrary that F is a fuzzy g-super closed g- super open set in X such that $A \leq F \leq 1- B$. Now, $F \leq 1- B$ implies $\rceil(F_q B)$. Therefore F is a fuzzy g- super closed g-super open set in X such that $A \leq F$ and $\rceil(F_q B)$. Hence $(X,\Im)$ is not fuzzy super GO-connected between A and B, which is a contradiction.

Sufficiency: Suppose contrary that $(X,\Im)$ is not fuzzy super GO-connected between A and B. Then $\exists$ an fuzzy g- super closed fuzzy g- super open F in X such that $A \leq F$ and $\rceil(F_q B)$.. Therefore F is a fuzzy g- super closed fuzzy g- super open set in X such that $A \leq F \leq 1- B$, which contradicts our assumption.

Theorem 2.3: If a fuzzy topological space $(X, \Im)$ is fuzzy super GO-connected between fuzzy sets A and B then A and B are non empty.

Proof: If the fuzzy set A is empty, then A is a fuzzy g- super closed fuzzy g- super open set in X and $A \leq B$. Now we claim that $\rceil(A_q B)$. If $A_q B$ then there exists an element $x \in X$ such that $A(x) + B(x) > 1$.It implies that $B(x) \geq A(x) > 1-B(x)$.Which contradicts our hypothesis. Hence $\rceil(A_q B)$ and $(X,\Im)$ is not fuzzy super GO-connected between A and B.

Theorem 2.4: If a fuzzy topological space $(X,\Im)$ is fuzzy super GO-connected between fuzzy sets A and B and $A \leq A_1$ and $B \leq B_1$ then $(X,\Im)$ is fuzzy super GO-connected between A_1 and B_1.

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Proof: Suppose $(X,\Im)$ is not fuzzy super GO-connected between fuzzy sets A_1 and B_1. Then there is a fuzzy g- super closed fuzzy g- super open set F in X such that $A_1 \leq F$ and $\rceil(F_qB_1)$. Clearly $A \leq F$. Now we claim that $\rceil(F_qB)$. If F_qB then there exists a point $x \in X$ such that $F(x) + B(x) > 1$. Now, $A(x) + B(x) > F(x) + B_1(x) > 1$, because, $F(x) \geq A(x)$ and $B(x) \geq B_1(x)$.Therefore F_qB_1.Which is a contradiction. Hence $(X,\Im)$ is not fuzzy super GO-connected between A and B.

Theorem *2.5*: A fuzzy topological space $(X,\Im)$ is fuzzy super GO-connected between A and B if and only if it is fuzzy super GO-connected between gcl(A) and gcl(B).

Proof: Necessity: Follows on utilizing theorem 2.4, because $A \leq gcl(A)$ and $B \leq gcl(B)$.

Sufficiency: Suppose $(X,\Im)$ is not fuzzy super GO-connected between A and B, Then there is a fuzzy g- super closed fuzzy g- super open set F of X such that $A\leq F$ and $\rceil(F_qB)$. Since F is fuzzy g- super closed and $A \leq F$, $gcl(A) \leq F$. Now, $\rceil(F_qB) \Rightarrow F \leq 1\text{-}B$. Therefore $F = gsint\ F \leq gsint\ (1\text{-}B) \leq 1\text{-}(gscl(B))$. Hence $\rceil(F_q\ gcl(B))$ and X is not fuzzy super GO-connected between gscl(A) and gscl(B).

Theorem2.6: Let $(X,\Im)$ be a fuzzy topological space and A and B be two fuzzy sets in X. If A_qB , then $(X,\Im)$ is fuzzy super GO-connected between A and B.

Proof: If F is any fuzzy g- super closed fuzzy g- super open set of X such that $A \leq F$, then
$A_qB \Rightarrow F_qB$

Remark 2.2: The converse of Theorem2.6 may not be true. For,

Example 2.2: Let X = {a, b} and the fuzzy sets U, A and B are defined as follows;U (a) =0.5 , U (b) =0.6 , A (a) =0.4 , A(b)=0.6 , B (a) =0.5 , B(b)=0.4
Let $\Im$ ={ **0,**U,1 } be a fuzzy topology on X .Then $(X, \Im)$ is fuzzy super GO connected between A and B but $\rceil(A_qB)$.

Theorem 2.7: A fuzzy topological space $(X,\Im)$ is fuzzy super GO-connected if and only if it is fuzzy super GO-connected between every pair of its non empty fuzzy sets.

Proof: Necessity: Let A, B be any pair of fuzzy subsets of X. Suppose $(X,\Im)$ is not fuzzy super GO-connected between A and B. Then there is a fuzzy g- super closed fuzzy g- super open set F of X such that $A \leq F$ and $\rceil(F_qB)$. Since A and B are non empty it follows that F is a non empty proper fuzzy g- super closed fuzzy g- super open set of X. Hence $(X,\Im)$ is not fuzzy super GO-connected.

Sufficiency: Suppose $(X,\Im)$ is not fuzzy super GO-connected. Then there exit a non empty proper fuzzy g- super closed fuzzy g- super open set F of X. Consequently X is not fuzzy super GO-connected between F and 1-F, a contradiction.

Remark 2.3: If a fuzzy topological space $(X,\Im)$ is fuzzy super GO-connected between a pair of its fuzzy subsets it is not necessarily that $(X,\Im)$ is fuzzy super GO-connected between every pair of its fuzzy subsets and so is not necessarily fuzzy super GO-connected.

Example 2.3: Let X = { a ,b } and U, A ,B and C be the fuzzy sets on X defined as follows:
U (a)=0.5, U (b) =0.6, A (a) =0.4, A (b)=0.6, B (a) =0.5, B (b)=0.4, C (a) =0.2, C (b)=0.3
Let $\Im$ ={ **0,**U,1 } be a fuzzy topology on X .Then $(X, \Im)$ is fuzzy super GO-connected between A and B but it is not fuzzy super GO-connected between B and C. Also $(X,\Im)$ is not fuzzy super GO-connected.

Theorem 2.8: Let $(Y,\Im_Y)$ be a subspace of a fuzzy topological space $(X,\Im)$ and A, B be fuzzy sets of Y. If $(Y,\Im_Y)$ is fuzzy super GO-connected between A and B then so is $(X,\Im)$.

Proof: Suppose contrary that $(X,\Im)$ is not fuzzy super GO- connected between A and B. Then there exists a fuzzy g- super closed g- super open set F of X such that $A \leq F$ and $\rceil(F_qB)$. Put $F_Y = F \cap Y$. Then F_Y is fuzzy g- super closed fuzzy g- super open set in Y such that $A \leq F_Y$ and $\rceil(F_{Yq}B)$. Hence $(Y,\Im_Y)$ is not fuzzy super GO-connected between A and B, a contradiction.

Theorem 2.9: Let $(Y,\Im_Y)$ be an fuzzy super closed open subspace of a fuzzy topological space $(X,\Im)$ and A , B be fuzzy subsets of Y. If $(X,\Im)$ is fuzzy super GO-connected between A and B then so is $(Y,\Im_Y)$.

Proof: If $(Y,\Im_Y)$ is not fuzzy super GO-connected between A and B then there exist a fuzzy g- super closed fuzzy g- super open set F of Y such that $A \le F$ and $\rceil(F_q B)$. Since Y is fuzzy super closed super open in X, F is a fuzzy g- super closed fuzzy g- super open set in X. Hence X cannot be fuzzy super GO-connected between A and B, a contradiction.

III. FUZZY SET GO-CONNECTED MAPPINGS

Definition 3.1: A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy set super GO-connected provided that f(X) is fuzzy super GO-connected between fuzzy sets f(A) and f(B) with respect to relative fuzzy topology if f(X) is fuzzy super GO-connected between A and B.

Theorem 3.1: A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy set super GO-connected if and only if $f^{-1}(F)$ is a fuzzy g- super closed fuzzy g- super open set of X for every fuzzy super closed fuzzy super open set F of f(X).

Proof: Necessity: Let F be a fuzzy closed fuzzy open set of f(X). Suppose $f^{-1}(F)$is not fuzzy g- super closed fuzzy g- super open set X. T hen X is fuzzy GO-connected between fuzzy set $f^{-1}(F)$ and $1\text{-}f^{-1}(F)$.Therefore f(X) is fuzzy super connected between $f(f^{-1}(F)$ and $f(1 - f^{-1}(F))$. But $f(f^{-1}(F) = F \cap f(X) = F$ and $f(1 - (f^{-1}(F)) = f(X) \cap (1\text{-}F) = 1\text{-}F$, implies that F is not fuzzy g- super closed fuzzy g- super open set in X , a contradiction.

Sufficiency: Let X be a fuzzy super GO-connected between fuzzy sets A and B .Suppose f(X) is not fuzzy super connected between f(A) and f(B).Then by theorem 2.2 there exists a fuzzy g- super closed fuzzy g- super open set F in f(X) such that $f(A) \le F \le 1\text{-}f(B)$ .By hypothesis $f^{-1}(F)$ is fuzzy g- super closed fuzzy g- super open set of X and $A \le f^{-1}(F) \le 1\text{-}B$. Therefore X is not fuzzy super GO-connected between A and B a contradiction. Hence f is fuzzy super set GO-connected.

Theorem 3.2: A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy set GO-connected if and only if $f^{-1}(F)$ is a fuzzy g- super closed fuzzy g- super open set of X for every fuzzy super closed fuzzy super open set of Y.

Proof: Obvious.

Theorem 3.3: Every fuzzy set connected mapping is fuzzy set super GO- connected.

Proof: Obvious.

Remark 3.1: The converse of the theorem 3.3 is not be true for;

Example3.1: Let X={a,b} ,Y= {p,q } and U(a) = 0.5 , U (b) = 0.6, V (p) = 0.2, V (q) = 0.3, W (p) =0.8, W (q) = 0.7 be the fuzzy sets Let $\tau=\{0,U,1\}$ and $\sigma = \{0,V,W,1\}$ be the fuzzy topologies on X and Y respectively. Then the mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is defined as follows f(a)=p, f(b)=q is fuzzy set super GO-connected but not fuzzy set super connected.

Theorem3.4: Every mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ such that f(X) is fuzzy super GO-connected is a fuzzy super GO-connected mapping.

Proof: Let f(X) be fuzzy connected. Then no non empty proper fuzzy set of f(X) which is fuzzy super closed and fuzzy super open. Hence f is fuzzy set super GO-connected.

Theorem 3.5: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a fuzzy set super GO-connected mapping. If X is fuzzy super GO- connected. Then f(X) is fuzzy super connected.

Proof: Suppose f(X) is fuzzy super disconnected. Then there is a nonempty proper fuzzy super closed super open set F of f(X) .Since f is fuzzy set super GO-connected by theorem 2.2, $f^{-1}(F)$ is a non empty proper fuzzy g- super closed g- super open set of X. Consequently X is not fuzzy super GO-connected.

Theorem 3.6: Let f: $X \rightarrow Y$ be a surjective fuzzy set super GO-connected and g: $Y \rightarrow Z$ is a fuzzy set super connected mapping. Then gof: $X \rightarrow Z$ is fuzzy set super GO-connected.

Proof: Let F be a fuzzy closed fuzzy open set of g(Y).Then $g^{-1}(F)$ is a fuzzy closed fuzzy open set of Y = f(X) . And so $f^{-1}(g^{-1}(F)$ is a g-closed g-open fuzzy set in X. Now (gof)(X) = g(Y) and $(gof)^{-1}(F)=f^{-1}(g^{-1}(F))$,by theorem 3.1 gof is fuzzy set GO-connected.

Theorem 3.7: Let f: $X \rightarrow Y$ be a mapping and g: $X \rightarrow X \times Y$ be the graph mapping of f defined by g(x) = (x, f(x)) for each $x \in X$. If g is fuzzy set GO-connected , then f is fuzzy set GO-connected.

Proof: Let F be a fuzzy closed fuzzy open set of the subspace f(X) of Y. Then X x F is fuzzy closed fuzzy open set of the subspace X x f(X) of the fuzzy product space X x Y. Since g(X) is a subset of X x f(X), (X x F) $\cap$ g(X) is a fuzzy closed fuzzy open set of the subspace g(X) of X x Y. By theorem 3.1,$g^{-1}((Xx\ F) \cap g(X)) = g^{-1}((XxF) = f^{-1}(F)$ that $f^{-1}(F)$ is a fuzzy g-closed fuzzy g-open set of X .Hence by the theorem 3.1, f is fuzzy set GO-connected.

Theorem 3.8: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a fuzzy weakly g-continuous surjective mapping, and then f is fuzzy set GO-connected.

Proof: Let V be any fuzzy closed open set of Y. Since f is fuzzy weakly g-continuous, $f^{-1}(V) \leq gint\ (f^{-1}(cl(V))) = gint(f^{-1}(V))$.And so $f^{-1}(V)$ is fuzzy g-open set of X. Moreover we have $gcl(f^{-1}(V)) \leq gcl(f^{-1}(int(V))) \leq f^{-1}(V)$.This shows that $f^{-1}(V)$ is fuzzy g-closed set of X. Since f surjective by the theorem 3.1, f is fuzzy set GO-connected mapping.

Defination3.2: A fuzzy topological space (X,τ) is said to fuzzy extremely disconnected if the closure of every fuzzy open set of X is fuzzy open in X.

Theorem 3.9: Let (Y,σ) be a fuzzy extremely disconnected space. If a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy set GO-connected then f is fuzzy almost g-continuous.

Proof: Let V is a fuzzy open set of Y .Then gcl(V) is a fuzzy closed open set in Y. Since f is fuzzy set GO-connected $f^{-1}(cl(V))$ is fuzzy g-closed g-open set of X. Therefore $f^{-1}(V)\leq f^{-1}(cl(V))\leq gint\ f^{-1}(cl(V))\leq f^{-1}(gint(cl(V)))$.Hence f is fuzzy almost g-continuous.

Corollary3.1: Let (Y,σ) be a fuzzy GO- extremely disconnected space. If a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy set GO-connected then f is fuzzy weakly g-continuous.

REFERENCES

[1] Azad K.K. , on fuzzy semi continuity fuzzy almost continuity and fuzzy weakly continuity, J. Math. .Anal. Appl. 82(1981),14-32.

[2] Chae G.I., Thakur S.S. and Malviya R. Fuzzy set connected functions, East Asian Math. J. 23(1)(2007),103-110.

[3] Chang C.L. Fuzzy topological spaces. J. Math. Anal. Appl. 24 (1968), 182-190.

[4] Fateh D.S. and Bassan D.D. Fuzzy connectedness and its stronger forms J. Math.. Anal. Appl. 111 (1985)449-464.

[5] Kwak J.H. set connected mappings Kyungbook Math. J. 11(1971), 169-172.

[6] Maheshwari S .N., Thakur S. S. and Malviya R. Connectedness between fuzzy sets, J. Fuzzy Math 1 (4) (1993),757-759.

[7] Mishra M.K. ,et all on " Fuzzy super closed set" International Journal International Journal of Mathematics and applied Statistics Vol-3,no-2(2012)143-146.

[8] Mishra M.K. ,et all on " Fuzzy super continuity" International Review in Fuzzy Mathematics ISSN : 0973-4392 July –December2012Accepted.

[9] Mishra M.K., Shukla M. "Fuzzy Regular Generalized Super Closed Set" Accepted for publication in International Journal of Scientfic and Research Publication ISSN2250-3153.December 2012.(Accepted).

[10] Noiri T. On set connected mappings Kyungpook Math. J. 16(1976), 243-246.

[11] Pu. P.M. and Liu Y.M. Fuzzy topology I. Neighborhood structure of a fuzzy point Moore-Smith convergence.J.Math.Anal.Appl.76 (1980), 571-599.

[12] Pu. P.M. and Liu Y.M. Fuzzy topology II. Product and Quotient spaces. J. Math. Anal. Appl.77(1980) 20-37.

[13] Thakur S. S. and Malviya R. Generalized closed sets in fuzzy topology, Math. Notae 38(1995) 137-140

[14] Thakur S. S. and Malviya R. Fuzzy gc-Irresolute Mappings , Proc. Math. Soc. BHU 11(1995),184-186

[15] Yalvac T.H. Fuzzy sets and functions in fuzzy spaces J. Math. Anal. Appl. 126 (1987), 409-423

[16] Zadeh L. A. Fuzzy sets Inform and control 6 (1965), 339-8-358.