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Mathematical Model by Fuzzy Rules from Dominating Graphs

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Abstract- Mathematical models constructed by fuzzy rules applying the gradient descent method have been widely used in many designs. Adjusting the fuzzy model built for the pre processed data from a dominating graph is the another view point approach of this work. By treating the triangular membership function as two disjoint ones mutual independence of fuzzy rules showed. In that way mathematical models constructed by fuzzy rules by applying gradient descent method with domination graphs.

Index Terms- Fuzzy modeling - domination graphs - Gradient descent method.

I. Introduction

Fuzzy rule base normally consists of a certain number of fuzzy rules. Depending on the types in the consequent parts, the fuzzy rules can be toughly classified in to the following briefly reviews those most commonly used fuzzy model. If x_1 is A_{i1} and x_2 is A_{i2} and …… and x_{in} is A_{in} , then y is w_i ,where w_i's are singletons. For this fuzzy model, the inferred output

$$W_c = \frac{\sum_{i=1}^{R} \mu_{iW_i}}{\sum_{i=1}^{R} \mu_i} \qquad (1)$$

Where μ_i represents the firing degree of the antecendent part in the i^{th} rule and R is the number of rules. Assume that the product operation is used, then

$$\mu_i = \prod_{j=1}^{n} Aij(xj) \qquad (2)$$

Where Aij(xj) represents, the membership degree for the data x_j on fuzzy sets Aij, the error function for given pattern as follows.

$$E=½[w_c – o_d]^2 \qquad (3)$$

Where O_d denotes output

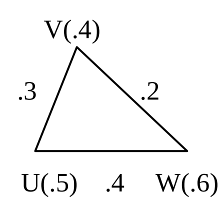
In this paper we will focus on investigating fuzzy models constructed by fuzzy rules only to which the gradient descent method can be successfully applied. The organization of this paper discussed how the raw data are pre processed and how the conventional gradient descent method is modified to handle the tuning job. To simplify the tuning process and to enhance the system performance, we suggest to use membership function dominating graph of fuzzy graph.

II. Preliminaries

Definition: A fuzzy set of a base set V is specified by its **membership function** σ where
σ : v -> [0,1] assigning to each u Є v the degree or grade to which u belongs to σ

Definition : A fuzzy graph G = (σ , μ) is a pair of function σ : V -> [0,1] and μ : v × v -> (0,1)where for all u, v Є V , we have μ (u, v) ≤ σ(u) ∩ σ(v) , μ is considered to be symmetric

Example:- A fuzzy graph G = (σ, μ) where
σ = {u/.5, v/.4, w/.6} and
μ ={(u, v)/.3, (u, w)/.4, (v, w)/.2}

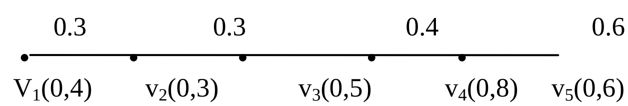


fig(1).

Definition: Let G=(σ,μ) be a fuzzy graph the arc (x,y) is a strong arc then the graph is called **strong arc fuzzy graph.**

Definition: Let G=(σ,μ) be a fuzzy graph and u be a node in G then there exists a node v, such that (u,v) is a strong arc, then u is said to be strong dominates v or simply dominates v.

If D is a fuzzy dominating set of a fuzzy graph G, then V-D need not be a fuzzy dominating set of G. For example, D={v_2, v_3, v_4} is a fuzzy dominating set of the fuzzy graph G given the following Fig(2). But its complement {v_1, v_5} is not a fuzzy dominating set of G.



Fig(2)

Definition: Let G=(σ,μ) be a fuzzy graph. A subset D of V is said of be **fuzzy dominating set of G** if for every V ∈ V-D, there exists u ∈ D such that u dominates v.

Definition: Minimum cardinality among all minimal fuzzy dominating set of G is called **fuzzy dominating number** of G and is denoted by γ (G).

Definition: Maximum cardinality among all minimal fuzzy dominating sets is called **upper fuzzy domination number** of G and is denoted by Γ (G).

A fuzzy dominating set D of a fuzzy graph G is called a **minimum fuzzy dominating set**,

if |D| = γ (G) and is denoted by γ - set.

III. GRADIENT DESCENT METHOD FOR THE MATHEMATICAL MODELS.

Fuzzy model needs further optimization to fit the control or identification requirement, For example, adjusting the centers membership function from a domination graph can affect the firing degrees of the fuzzy rules. Assuming the parameters to be adjusted in fuzzy models are P_j's. The quantity of P_j to be adjusted in each iteration can be obtained by taking the derivatives of $\frac{\partial E}{\partial Pj}$ from(3). As long as the quantity of P_j to be adjusted is determined the new P_j can be updated as follows.

$$P_j(t+1) = P_j(t) - \gamma_p \frac{\partial E}{\partial Pj} \qquad (4)$$

Where γ_p is the prearranged learning rate.

For example, the consequent parts w_i's in fuzzy rules can be updated

$$W_i(t+1) = w_i(t) - \gamma_w \frac{\partial E}{\partial Wi}$$

$$= w_i(t) - \gamma_w \frac{\partial E}{\partial Wc} \frac{\partial Wc}{\partial Wi}$$

$$= w_i(t) - \gamma_w (w_c - o_d)\, \mu_i \Big/ \sum_{i=1}^{R} \mu_i$$

Where γ_w represents the prearranged learning rate. Similarly, if the triangular member ship functions are used taking first derivatives of the error function with respect to the centers and spreads of the membership function allow us to systematically adjust the membership functions. The purpose of preprocessing the original data is to reallocate the distribution of data to simplify the fuzzy modeling and in turn to improve the system performance.

IV. RESULT- MEMBERSHIP FUNCTIONS OF DOMINATION GRAPHS.

We propose a simple but effective method to improve the inference outcome from the preprocessed model. A membership function is still shared by the data located in two consecutive intervals. This implies that adjusting a rule to satisfy a specific pattern will exert some effects on the neighbouring data. A continuing refinement must be done to make a compromise.

A fuzzy rule for the single-input-single-output (SISO) system is originally created like. If x is A_i then y is w_i .The data in the same region to have their own fuzzy rules and to simplify the adjustment of consequent real numbers, each membership function is virtually bifurcated at the center into two separate ones.(ie) The left and right parts.

A fuzzy rule is separate into two rules as follows.

If x is A_{iL} then y is w_{iL}, If x is A_{iR} then y is w_{iR}

Where A_{iL} and A_{iR} represent the left and right parts of the membership function A_i respectively, as shown in fig F(x). A fuzzy model with each membership function virtually treated as two individual ones. Of course, increasing the rule numbers will enlarge the rule base and in turn complicate the inference process.

Since the membership function for the premise part are equally distributed in the transformed domain each input pattern will activate at most fuzzy rules. Except the leftmost and the rightmost intervals which have only one fuzzy set being mapped, each transformed data will be mapped to two non-zero membership degrees and the sum of both is exactly equal to one. This implies that Equation(1) can be further simplified.

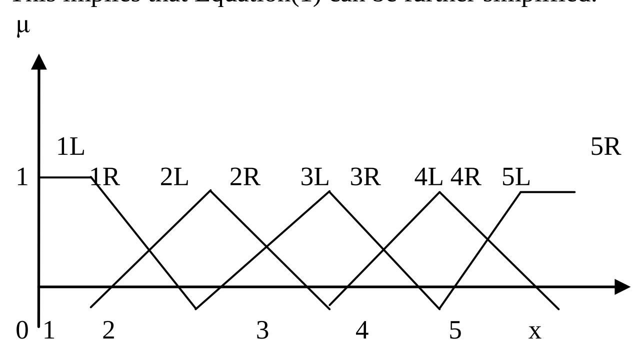


Fig. F(x) membership functions of input variable SISO System.

$$W_c = \frac{\sum_{i=1}^{2} \mu_{iW_i}}{\sum_{i=1}^{2} \mu_i}$$

$$= \sum_{i=1}^{2} \mu_i\, w_i = \mu_1\, w_1 + (1-\mu_1)\, w_2$$

Assume that there are n transformed data patterns locating in the same interval. Since those n patterns share the same fuzzy rules, the consequent parts in those rules must satisfy those patterns in a well refined fuzzy model. We can compare the adjusted consequent singletons of fuzzy rules with those desired ones. The desired consequent real numbers can be calculated by the least squared method. As follows

$$\begin{bmatrix} O_{d1} \\ O_{d2} \\ \cdot \\ \cdot \\ \cdot \\ O_{dn} \end{bmatrix} = \begin{bmatrix} \mu_1 & 1-\mu_1 \\ \mu_2 & 1-\mu_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mu_n & 1-\mu_n \end{bmatrix} \begin{bmatrix} w_i \\ w_{i+1} \end{bmatrix}$$

$$O = PW \qquad (5)$$

For simplicity, we can denote Equation(5) as follows.

Where

$O = [Od_1, Od_2 - - - - - - - -Od_n]^T$

$W = [w_i , w_{i+1}]^T$ and

$$P = \begin{bmatrix} \mu_1 & 1-\mu_1 \\ \mu_2 & 1-\mu_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \mu_n & 1-\mu_n \end{bmatrix} \qquad (6)$$

Equation(6) is created to represent the vector of desired inputs.

Using the pseodomverse of P, We can calculate the consequent singleton vector W as follows

$$W = (P^{-1} P)^{-1} \; P^{-1} O$$

V. CONCLUSION

The gradient descent technique was successfully applied to adjusting fuzzy models as mathematical models by fuzzy rules from dominating graphs.

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