Construction of B-Algebra on Cycles of Aunu Permutation Pattern

*** M. A. Mungadi [1] A.A. Haliru [1] S. Abdulrahman. [2] and M. S. Magami [3]**

[1]Department of Mathematics Kebbi State University of Science and Technology, Aliero P.M.B. 1144 Aliero, Kebbi State.

[2]Department of Mathematics Federal University Birnin Kebbi , Kebbi State.
[3]Department of Mathematics,Usmanu Danfodiyo University,P.M.B. 2346 Sokoto, Nigeria.

*Corresponding Author: maliyumunggadi19@gmail.com

DOI: 10.29322/IJSRP.9.01.2019.p8598
http://dx.doi.org/10.29322/IJSRP.9.01.2019.p8598

Abstract

Application of some algebraic properties of B-Algebra in AUNU Permutation Patterns was developed. Then the cycles AUNU Group were applied to form a table that satisfied some properties of B-algebra and some relevant theorem that will back our constructions was proposed. We also applied B-algebra in graph theory were we obtained a directed graph its adjency matrix

Keywords: *AUNU Groups, AUNU Patterns, B-Algebra, Permutations, AUNU Number and Graph*.

1. Introduction

AUNU permutation pattern are a class of (123) and (132)-avoiding permutation pattern with special properties associated with succession schemes. These patterns have been used extensively in formulation of different structures such as group, graph and in study of different algebra schemes such as cyclic designs and circuit design see Mustapha and Ibrahim, (2013); Ibrahim, (20005); Magami *et al*, (2012). Some other theoretic properties of the AUNU Pattern and AUNU group were identify and discussed especially in relation to integer modulo group see Usman and Ibrahim, (2011). Ibrahim and Saidu, (2016.) Present non-associative and non commutative properties of 123-avoiding pattern of AUNU permutation pattern. The research describe how non associative and non commutative properties can be established by using the Cayley table on which a binary operation is defined to act on the 123-avoidind and 132-avoiding pattern of AUNU permutation using a pairing scheme. The result have generate larger matrix from permutation of point of the AUNU Pattern of prime cardinality see

2. Some Basic Definition

2.1. B**-algebra-:** A B-algebra is a non-empty set X with constant 0 and binary operation "*" satisfying the following axioms.

B1 $x * x = 0$

B2 $x * 0 = x$

B3 $(x * y) * z = x * (z * (0 * y))$

For all x, y and z in X

2.2. Sub Algebra: -Let $(X,*,0)$ be a B-algebra. A non empty subset N of X is said to be sub algebra if $x * y \in X \ \forall \ x, y \in N$

2.3. Commutative: - A B-algebra$(X,*,0)$ is said to be commutative if $a * (0 * b) = b * (0 * a) \forall \ a, b \in X$ otherwise it is not commutative.

2.4. Center of *X*: - let $(X,*,0)$ be a B-algebra define $Z(X) = \{x \in X | x * (0 * y) = y * (0 * x) \ \forall \ x, y \in X\}$we call it center of X.

Note that $0 \in Z(X)$

2.5. B-algebra $(X,*,0)$ is **cyclic B-algebra** if there exist $x \in X$ such that $X = \langle x \rangle_B$. the B-algebra $(z, -, 0)$ is cyclic since$X = \langle 1 \rangle_B$.

2.6. AUNU Numbers: - There are two types of AUNU Numbers; the (123)-avoiding class obtained from a recursion relation as follows:

$$N(A_n(123)) = \frac{P_n - 1}{2}$$

Give rise to: 2, 3, 5, 6, 8, 9, 11, 14 …

Corresponding to the length of 5, 7,11,13,17, 19 …

The sequence 2, 3, 5, 6, 9, 11, 14… is called the AUNU numbers corresponding to the (123)-avoiding class of permutation.

On the other hand the (132)-avoiding class of AUNU permutation patterns is obtained from a relation (Ibrahim 2004, Ibrahim 2006) as follows:

$$N\left(A_n(132)\right) = n + (m - 1), \ m \leq n. \text{ and} n, m \geq 3$$

Give rise to: 5,7,9,11,13 …

Corresponding to the length of 3,4,5,6, 7 …

The sequence 5, 7, 9, 11, 13… is called the AUNU numbers corresponding to the (132)-avoiding class of permutation

Where $N(A_n(123))$ is the number of the class of numbers expressed as permutations, that avoid (123) patterns while P_n is the n^{th} prime number n ≥ 5.

Notation and Method of Application

Throughout this reseach we shall adopt the use of the following basic notations and symbols for the purpose of both theoretical derivations and in establishing some fundamental result arising there from.

Let $\Omega_{(132)} = \{n + (m - 1)\}$ for $\{m \leq n \in \mathbb{Z}^+\}$and $n = \{3,4,5, \ldots p\}$

Then,

$$\Omega_{(132)} = \{5,7,9, \ldots p\}$$

Is called AUNU number of the 132-avoiding pattern

Let the partition of $\Omega_{(132)} * \Omega_{(132)} \in Z_p$ where Z_p is the integer modulo p and the binary operation $*$ donates addition and subtraction.

Then $\Theta: \Omega_{(132)} * \Omega_{(132)} \rightarrow Z_p$ such that $|\Theta| = x$ where Θ defines a mapping from $\Omega_{(132)} * \Omega_{(132)}$ to Z_p such that Z_p restrict elements of $\Omega_{(132)} * \Omega_{(132)}$ to integer modulo$p$, for some prime number$p \geq 5$. It follows that $|\Theta: \Omega_{(132)} * \Omega_{(132)} \rightarrow Z_p| = x$, where in this case, $x \in Z^+$.

3 Application in AUNU Permutation Pattern

Proposition:

Let $\Omega = \{0,1,2,3,4\} \subset Z_5$ be a cycle of AUNU group of order p, were $p \geq 5$ under binary operation "$*$" then $\Omega_{132} * \Omega_{132}$ is a B-Algebra under $*$.

Proof

For a $\Omega_{132} * \Omega_{132}$ to be a B-Algebra it must satisfy at least two condition of B-Algebra

Then, regarding $\{0,1,2,3,4\}$ as an element to construct tables satisfies some properties of B-algebra.

Table 1: operation table of cycle

$*$	**0**	**1**	**2**	**3**	**4**
0	0	0	0	0	0
1	1	0	0	0	0
2	2	1	0	0	0
3	3	2	1	0	0

ISSN 2250-3153

4	4	3	2	1	0

Hence the zero elements represent diagonal element also the table form lower triangular matrix

Then $(\Omega; *; 0)$ is a B-algebra.

Theorem

Let (Ω,*, 0) be an AUNU number of the (132) and (123)-avoiding pattern with binary operation

''*'' and constant 0 then **Ω** is a B-algebra satisfying the following condition.

$$B1.\ x * x = 0$$

$$B2.\ x * 0 = x$$

$$B3.\ (x * y) * z = x * \big(z * (0 * y)\big), \forall\, x, y, z \in \Omega$$

Proof

For any $x, y, z \in \Omega$

If $x * z = y * z \Rightarrow x = y$ means that the element x and y are unique in Ω.

Therefore

Absorption law

Uniqueness of identity

$$x * x = 0 \Rightarrow x = -x$$

$$x + x = 0$$

$$x = -x$$

$$x + (-x) = 0$$

From condition 2 of B-algebra

$$x * 0 = x$$

$$y * 0 = y$$

$$x * 0 = y * 0 \Rightarrow x = y$$

From condition 3 of B-algebra

$$(x * y) * z = x * \big(z * (0 * y)\big)$$

$$= x * (z * 0) * y$$

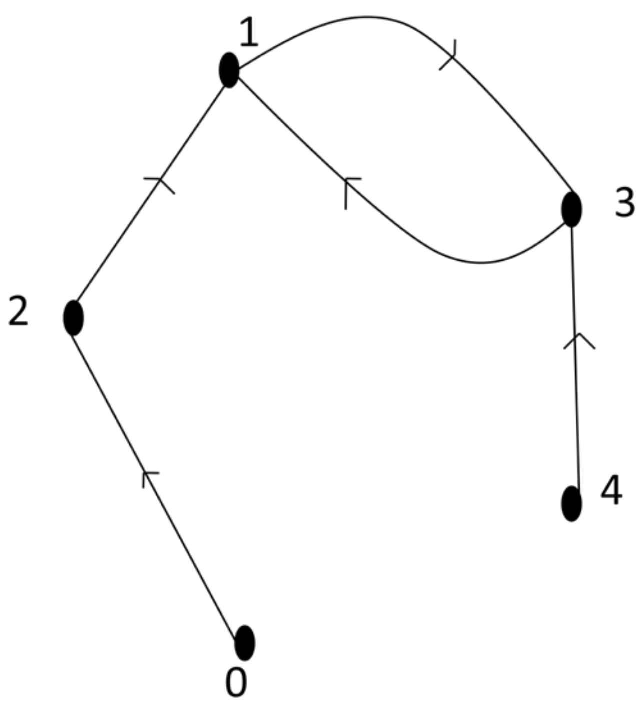
$$= x * \Big(y * \big(0 * (0 * z)\big)\Big)$$

$$= x * (y * z)$$

Therefore the RHS yield the LHS and result follows. By the absorption law that is $ax = ay \Rightarrow x = y$

4. B-Algebra in Graph Theory

Example 1. Suppose a function g is defined on B-algebraΩ, that is $\Omega = \{1,2,3,4,0\}$ by $g(x) = x^2 + 5x^3 + 2\ \forall x \in \Omega$. then the following pairs of point were obtained from the function $V(G) = \{(0,2),(1,3),(2,1),(3,1),(4,3)\}$

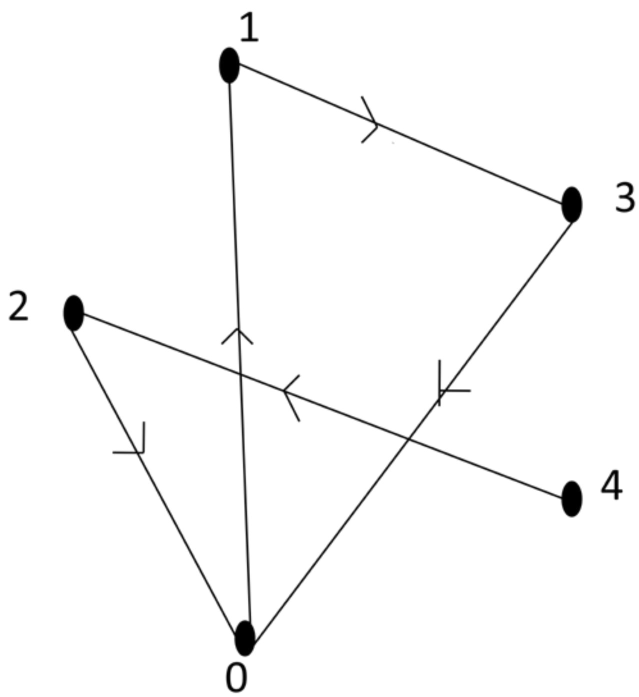


Graph 1

Adjency matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Example 2. Suppose a function g is defined on B-algebraΩ, that is $\Omega = \{1,2,3,4,0\}$ by $g(x) = x^2 + 2\ \forall x \in \Omega$. then the following pairs of point were obtained from the function $V(G) = \{(0,1),(1,3),(2,0),(3,0),(4,2)\}$



Graph 2

Adjency matrix

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Conclusion

This construction has explored the application of AUNU Permutation Pattern, were the cycle of (132) - avoiding pattern was used in table 1which satisfied properties of B-algebra, the zero element in the table represent diagonal element and again table form a lower triangle matrix. The B-algebra was applied in graph were a directed graph was obtained see graph 1 &2 and their adjency matrix.

REFERENCES

Ibrahim, A. A. and Sa'idu, I. A. (2016). Non Association Property of 123-Avoiding Class of Aunu Permutation Pattern, *Advances in Pure Mathematics*, 6, 51-57

Ibrahim, A. A. (2005b). On the Combinatorics of Succession in a 5 Element Sample. *Abacus Journal of the Mathematical Association of Nigeria;* 32, 410-415

Magami, M. S. and Ibrahim, A.A. (2012). Construction of Association Scheme Using Some (132)-Avoiding Class of Aunu Permutation Patterns. *Abacus Journal of Mathematical Association of Nigeria of Mathematics Association of Nigeria;* 39(2), 165-172.

Mustapha, A. and Ibrahim, A. A. (2013). Implementation of Aunu Permutation Pattern using Computer Algebraic System. *Computational Approach, Physical Science Research International*; l1(3), 54-61

Usman, A. and Ibrahim, A. A. (2011). A New Generating Function for Aunu Pattern: Application in Integer Group Modulo n. *Nigerian Journal of Basic and Applied Sciences;* 1, 1-4