# LIFO policy for Perishable Inventory systems under fuzzy environment

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***Abstract-*** This paper studies the performance characteristic of perishable inventory system with Last In First Out selection policies in Fuzzy environment. In our previous paper [1] we have discussed the performance characteristic with FIFO selection policies. Considering the same conditions as in [1] we have studied the spoilage rate, loss rate, mean time between stock outs inventory level and the distribution of age of items delivered.

***Index Terms***- Perishable inventory, first in first out, Fuzzy Poisson arrival, fuzzy demand rate, fuzzy shelf life time, customer separation time, the forward recurrence time.

## I. Introduction

Today a successful inventory control is recognized today as the key to maintain competitive market condition. The traditional motivation behind the inventory is to ensure compliance with customer demand and to guard against uncertainties arising in demand fluctuation and delivery lead time. Perishable items are one which may not lose its value or utility over time. Many retail products, such as food items, Pharmaceutical cut flowers etc have short life time. Not only these product generates a sustainable amount of revenue themselves they also derive store traffic. Despite its important the management of perishable is challenging. It require proper handling and storage and the use of right technology throughout the entire supply chain.

Excellent literature review of inventory models has a long history. Typical examples of items with limited shelf life include fresh products, drugs, chemical and films(Nahamias (1989)). Several studies are devoted to the management of P.I. S(Peterson and Silver (1979). In the grocery and pharmaceutical industry, expiration is responsible of 19% and 31% of total unsalable respectively (Joint Industry Unsalable Benchmark Survey,2003). Furthermore, Lystad *et al.* (2006) reported that about $30 billion are lost due to perishability in US grocery industry. Another investigation in Nordic retail sector (Karkkainen,2003), reported that the spoilage costs of perishables are up to 10 percent of total sales.

The amount of perishable goods thrown away by retailers is alarmingly high and has come under continuous public scrutiny in recently years (Bloom 2010, Stuart 2009). According to a recent study by Friends of the Earth, the four main supermarkets in Hong Kong throw away 87 tons of foods per day, most of which end up in lands (Wei 2012). Therefore to better match the supply and demand many retailers have used clearance sales as a strategy to sell items approaching their expiration dates at a reduced price. But it is clear that most probably t there is typically no coordination between clearance sales and replenishment. So a study on such type of inventory models becomes essential.

This study first characterize the stock dynamics needed to evaluated various performance measures for LIFO policy. Using these results one can choose the desired replishment rate and could use to consider the purchase, service level, tax and inventory holding cost.

## II. The Model Description

**Notations:**

- $\tilde{\lambda}$- fuzzy arrival rate
- $\tilde{\mu}$-fuzzy service rate
- $\tilde{D}$- fuzzy shelf life time
- $\tilde{T}_{Cj}$ - customer separation time of the jth customer
- $\tilde{T}_F$ - the forward recurrence time of the time $\tilde{T}_C$
- $\tilde{X}$ **(t)** - the anticipated time to selection *of an item* if no other items were to come, given that at time **t** that item has not been selected.

**2.1 Assumptions:**

The perishable inventory system of interest has the following characteristics:

- The arrival of fresh items follows a Poisson process with mean $\tilde{\lambda}$. arrivals per unit time.
- The mean demand rate is $\tilde{\mu}$ requests per unit time.
- Each demand request is for one unit at a time.
- Demand requests arriving when the inventory system is empty are lost.
- The stored items have a shelf life of $\tilde{D}$ time units.
- The demand process is a Poisson process independent of the arrival process.
- An item which is not used to meet a demand request within $\tilde{D}$ time units perishes (exits the system).

## III. THE LIFO INVENTORY SYSTEM

Item arrive to inventory one at time in a Poisson rate $\tilde{\lambda}$. One has a sequence of demand epochs with iid customer separation time $\tilde{T}_{Cj}$ and associated demand rate $\tilde{\mu} = \frac{1}{E[\tilde{T}_{Cj}]}$. At each demand epoch if the inventory is not empty the freshest item is chosen. Each item has an non- deterministic shelf life time $\tilde{D}$ time units.

To analyze this system caution is needed. Most of the processes describing the system are not Markov. The number of items in storage, for example is not Markov. To make it fuzzy Markov, the process must be supplemented by the ages of all items in inventory and analysis is then too difficult.

### 3.1 The loss process $\tilde{X}(t)$

The following loss process $\tilde{X}(t)$ is Fuzzy Markovian and provides a stepping stone to the analysis of the FIFO system. At $\tilde{t}$ = 0, a fresh item with infinite shelf life arrives. Let $\tilde{X}(t)$ be the anticipated time to selection *of an item* if no other items were to come, given that at time $\tilde{t}$ that item has not been selected. The process terminates when the item is selected.

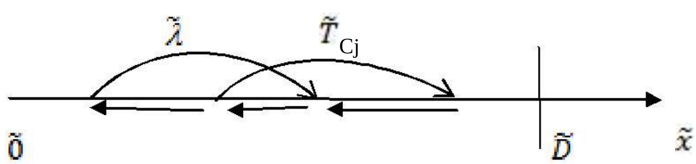
**Theorem 3.1:**

The loss process $\tilde{X}(t)$ is Markov.

**Proof:**

Consider the motion of $\tilde{X}(t)$ on Its state space $\tilde{N}$ is the union $\tilde{N} = \{\tilde{S} \oplus \tilde{E}\}$ of the set $\tilde{S} = \{(\tilde{s}, \tilde{x}) : 0 < \tilde{x} < \tilde{D}\}$ where stock is available and of the point state $\tilde{E}$ for stock out.. This motion may be described as follows: At t = 0, $\tilde{X}(t)$ = (0,0,0) is distributed as the forward recurrence time $\tilde{T}_F$ of the time $\tilde{T}_C$ between demand epoches, $\tilde{X}(t)$ decreases at unit rate between new arrivals. There is a constant hazard rate $\tilde{\lambda}$ for new arrivals of fresh items. Each new arrival delays the selection of the original item by a new independent random amount $\tilde{T}_{Cj}$, i.e. $\tilde{X}(t)$ increases by $\tilde{T}_{Cj}$. The process $\tilde{X}(t)$ is therefore fuzzy Markovian.. §§

The motion of $\tilde{X}(t)$ on the state space $N$ is shown in Figure 1 for the case of finite shelf life D.




Motion on the state-space $\tilde{N}$.

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Clearly, when the shelf life is infinite, the distribution of the time a new item spends in inventory is equal in distribution to the duration of a special fuzzy busy period BP* for a single server queueing system M/G/1 with arrival rate $\tilde{X}$, service time distributed as $\tilde{T}_C$, and initial backlog that of the forward recurrence time of $\tilde{T}_C$.

When the shelf life is finite with maximal life $\tilde{D}$, the time in system $\tilde{T}_{SD}$ is given instead by

$\tilde{T}_{SD}$ = min [BP*, $\tilde{D}$] , (1) and the distribution of $\tilde{T}_{SD}$ is evaluated below. When $\tilde{D}$ is very large one needs $\tilde{\lambda} < \tilde{\mu}$ to assure system stability. For $\tilde{D}$ being finite**,** the idle state is positive recurrent and the system is always stable due to the outdating of excess stock.

**Fundamental performance relationships**

It will be assumed in all that follows that $\tilde{T}_C$ is exponentially distributed with mean $\frac{1}{\tilde{\mu}}$. The forward recurrence time then has the distribution of $\tilde{T}_C$ and BP*, and the familiar busy period BP then coincide in distribution. The spoilage rate of arriving items is

$$\text{Spoilage Rate} = \tilde{\lambda}\,\tilde{P}\,[BP > \tilde{D}]. \quad (1)$$

The probability of spoilage for an individual item is

$$\tilde{P}\,[\text{Spoilage}] = \tilde{P}\,[BP > \tilde{D}]. \quad (2)$$

We assume that demand epochs taking place when the system is empty are ignored and the
customers are lost. In order to get the rate of lost sales we denote by $\tilde{e}_{\infty}$the probability of having an empty stock.

Getting $\tilde{e}_{\infty}$, we first note that the long term sales rate from the on-hand inventory is the demand rate minus the lost sales rate. This long term sales rate must equal the arrival rate minus the spoilage rate. Consequently, we get the following *inventory balance equation*

our perishable inventory systems:

$\tilde{\lambda}$ - $\tilde{\mu}$ = spoilage rate - lost sales rate = spoilage rate - $\tilde{\mu}\,\tilde{e}_{\infty}$ . (3)

Thus one has

**Spoilage Rate** = $\tilde{\lambda}$ - $\tilde{\mu}$ (1-$\tilde{e}_{\infty}$) = $\tilde{\lambda}$ $\tilde{P}$ [BP > $\tilde{D}$]. (4)

Similarly

**Loss Sales Rate** = $\tilde{\mu}\,\tilde{e}_{\infty}$ = $\tilde{\mu}$ +$\tilde{\lambda}$ -$\tilde{\lambda}$ $\tilde{P}$ [BP > D], (5)
and

$$\tilde{e}_{\infty} = \frac{\tilde{\mu}+\tilde{\lambda}-\tilde{\lambda}\tilde{P}\,[\mathrm{BP}>\mathrm{D}]}{\tilde{\mu}} \quad (6)$$

**Property 3.1** : When $\tilde{\mu}$ -> 0, it will be seen subsequently that $\tilde{e}_{\infty} \rightarrow e^{-\tilde{\lambda}\tilde{D}}$ this is *true for any selection policy* when there is no demand.

The inventory system **service level,** or the probability that stock is on hand, is
**Service Level** = 1-$\tilde{e}_{\infty}$

**E[number of items in system]** = $\tilde{\lambda}$ E[$\tilde{T}_{SD}$] (7)

**Lemma 3.2:** The expected time between stockouts E[$\tilde{T}_G$] is given by

$$\mathrm{E}[\tilde{T}_G] = \frac{1}{\tilde{\lambda}}\left(\frac{1}{\tilde{e}_{\infty}} - 1\right) \quad (8)$$

**Proof:** We use the following ergodic argument: Periods of stockout alternate with periods of stock availability. The stockout periods are exponentially distributed with mean duration equal to $\frac{1}{\tilde{\lambda}}$ . It follows that the alternating time intervals should satisfy:

$$\tilde{e}_{\infty} = \frac{\frac{1}{\tilde{\lambda}}}{\frac{1}{\tilde{\lambda}}+\mathrm{E}[\tilde{T}\mathrm{G}]} \quad (9)$$

This leads the desired result above. §§.

**Property** 3.2: The expected time between stockouts satisfies E[$\tilde{T}_G$] --> $\tilde{D}$ as $\tilde{\mu} \rightarrow \tilde{\lambda}$

**Property** 3.3: The duration of the stockout period is of course exponentially distributed with mean $\frac{1}{\tilde{\lambda}}$ because there are no backorders.

**Evaluation of the busy period distribution**

In order to get the distribution of the time in system TSD we need to evaluate the busy period distribution of the analogous single server queueing system M/M/1 described above.

It is known (Takacs [1962] ) that for M/M/I1 the busy period pdf is given by

$$\tilde{S}_{BP}(X) = 2\tilde{\mu}\, e^{-[(\tilde{\mu}+\tilde{\lambda})\,\tilde{x}]}\,\frac{I_1\left(2\tilde{x}\sqrt{(\tilde{\mu}\tilde{\lambda})}\right)}{2\sqrt{(\tilde{\mu}\tilde{\lambda})}\,\tilde{x}} \quad (10)$$

where $I_1$(.) is the modified Bessel function of the first kind and order 1.

This expression can be further simplified to obtain :

$$\tilde{S}_{BP}(Y) = 2\tilde{\mu}\, e^{-[(\tilde{\xi})\,\tilde{y}]}\,\frac{I_1(\tilde{y})}{\tilde{y}} \quad (11)$$

where

$$\tilde{y} = 2\sqrt{(\tilde{\mu}\,\tilde{\lambda})}\,\tilde{x} \quad (12)$$

and

$$\tilde{\xi} = \frac{[\tilde{\lambda}-\tilde{\mu}]^2}{2\sqrt{(\tilde{\mu}\,\tilde{\lambda})}} \quad (13)$$

$\tilde{S}_{BP}$ (X) *is valid for both* $\tilde{\lambda}<\tilde{\mu}$ *and* $\tilde{\lambda} > \tilde{\mu}$

We note that (3.11) will have

$$\tilde{P}\,[\mathrm{BP} < \infty] = \begin{cases} 1, & \text{if } \tilde{\lambda} < \tilde{\mu} \\ \frac{\tilde{\mu}}{\tilde{\lambda}}, & \text{if } \tilde{\lambda} > \tilde{\mu} \end{cases} \quad (14)$$

If a high service level is wanted, one needs to have $\tilde{\lambda} > \tilde{\mu}$. In that case one gets $\tilde{\xi}$ < 1.

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