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Poisson-Size-biased Lindley Distribution

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Abstract:
In this paper the general concept of Poisson-size-biased Lindley (PSBL) distribution is presented. Its p.m.f. is obtained. Some of its properties and the expressions for raw and central moments, coefficients of skewness and kurtosis are derived. The moment equations and the maximum likelihood estimators of the parameter of this Poisson size-biased Lindley (PSBL) distribution have been obtained for estimation its parameters. A simulation of a parameter is study has been proposed.

Key Words:
Size-biased Poisson-Lindley distribution; moment's equation estimation; maximum likelihood estimators;

I. INTRODUCTION

D. V. Lindley [2], has introduced a one-parameter distribution, known as Lindley distribution, given by probability density function (p.d.f.):

$$\mathrm{f}(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}\ ;\ x>0,\qquad \theta>0,\dots\dots(1.1)$$

M. Sankaran [10] has introduced one parameter Poisson-Lindley distribution (PLD) to model count data with probability mass function (p.m.f.):

$$f_0(x;\theta)=\frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}\ ,x=0,1,\dots.;\ \theta>0,\dots.(1.2)$$

The distribution arises from the Poisson distribution when its parameter λ follows a Lindley distribution with probability density function (p.d.f.)

$$\mathrm{g}_0(\lambda;\theta)=\frac{\theta^2}{\theta+1}(1+\lambda)e^{-\theta\lambda}\ ,\qquad \lambda>0,\qquad \theta>0,\dots\dots(1.3)$$

R. Shanker, S. Sharma and R. Shanker [16] proposed a two-parameter Lindley distribution of which the one-parameter Lindley distribution (LD) is a particular case, for modeling waiting and survival time's data. R. Shanker and A. Mishra [11] proposed a two-parameter Quasi Lindley Distribution (QLD) and studies its properties. It is found that in all data-sets the QLD provides closer fits than those by the Lindley distribution.

R. Shanker, S. Sharma and R. Shanker [15] proposed a discrete two parameter Poisson Lindley distribution (PLD), of which the M. Shankaran's [10] Poisson-Lindley distribution is a special case. It is derived by compounding a Poisson distribution with the discrete two-parameter Lindley distribution of R. Shanker,S. Sharma and R. Shanker [13]. They derived first four moments of this distribution and have discussed the estimation of the parameters by the moments. They have found that the two-parameter PLD is better fit and more flexible than the Shankaran'a one-parameter PLD to some data sets.

M. E. Ghitnany and D. K. Al-Mutairi [9] discussed estimation methods for the discrete Poisson Lindley distribution (1.2) and its applications. They derived a discrete two-parameter Poisson Lindley distribution by compounding a Poisson distribution with a two-parameter Lindley distribution obtained by R. Shanker, S. Sharma and R. Shanker [13].

In many a situation experimenters do not work with truly random sample from the population, in which they are interested, either by design or because of the fact that in many situations it becomes impossible to have random sample from the targeted population. However, since the observations do not have an equal probability of entering the sample, the resulting sampled distribution does not follow the original distribution. Statistical models that incorporate these restrictions are called weighted models. When an investigator records an observation by nature according to certain stochastic model, the recorded observation will not have the original distribution unless every observation is given an equal chance of being recorded. For example, suppose that the original observation x_0 comes from a distribution with p.m.f./p.d.f. $f_0(x_0)$ and that observation x is recorded according to a probability re-weighted by a weight function w(x) > 0, then x comes from a distribution with p.m.f./p.d.f.

$$f(x)=\frac{w(x)}{E[\,w(X_0)\,]}f_0(x)\dots\dots\dots\dots(1.4).$$

C. R. Rao [3] introduced distributions of this type and called them weighted distributions. The weighted distribution with w(x) = x is called size-biased/length-biased distribution. G. P. Patil and C. R. Rao [4] examined some general models leading to weighted distributions and showed how the weight w(x) = x occurs in a natural way in many sampling problems. A study of size-biased sampling and related form-invariant weighted distributions was made by G. D. Patil and J. K. Ord [5]. A survey of real-life applications of size-biased distributions may be found in G. D. Patil and C. R. Rao [3] and [4].

M. E. Ghitany and D. K. Al-Mutairi [8] proposed Size-biased Poisson-Lindley distribution and suggested its application. They consider the size-biased version of Poisson-Lindley distribution and obtained the p.m.f. of size-biased Poisson-Lindley (SBPL) distribution as

$$f(x;\theta) = \frac{x}{\mu_0} f_0(x;) = \frac{\theta^3}{\theta+2}\frac{x\,(x+\theta+2)}{(\theta+1)^{x+2}}, \qquad x = 1,2,\ldots;\ \theta > 0, \ldots (1.5)$$

Where, $\mu_0 = \frac{\theta+2}{\theta(\theta+1)}$ is the mean of the Poisson-Lindley distribution with p.m.f. (1.2). The SBPL distribution also arises from the size-biased Poisson (SBP) distribution with p.m.f.

$$\mathrm{g}(x/\lambda) = e^{-\lambda}\frac{\lambda^{x-1}}{(x-1)!}, \qquad x = 1,2,\ldots\ldots;\ \lambda > 0, \ldots\ldots (1.6)$$

when its parameter λ follows a size-biased Lindley (SBL) model with p.d.f.

$$\mathrm{h}(\lambda;\theta) = \frac{\theta^3}{\theta+2}\lambda(1+\lambda)e^{-\theta\lambda}, \qquad \lambda > 0, \theta > 0, \ldots\ldots (1.7)$$

Combining the equations (1.6) and (1.7) then the result will be:

$$f(x;\theta) = \int_0^\infty \mathrm{g}(x/\lambda)\ .\ \mathrm{h}(\lambda;\theta)\, d\lambda$$

$$= \frac{\theta^3}{\theta+2}\left\{\frac{x(x+\theta+2)}{(\theta+1)^{x+2}}\right\}, \qquad x = 1,2,\ldots.;\ldots (1.8)$$

This is same as equation (1.5). Thus it is clear that the size-biased version of Poisson Lindley distribution is same as that obtained by compounding size-biased Poisson and size-biased Lindley distributions. The mean (μ), variance (σ^2), coefficient of skewness ($\sqrt{\beta_1}$) and coefficient of kurtosis (β_2) for the SBPL distribution proposed by M. E. Ghitany and D. K. Al-Mutairi [9] are as:

$$Mean(\mu_1) = \mu_1' = \frac{\theta^2+4\theta+6}{\theta(\theta+2)},$$

$$Variance(\mu_2) = \frac{2(\theta^3+6\theta^2+12\theta+6}{\theta^2(\theta+1)^2},$$

$$\sqrt{\beta_1} = \frac{\theta^5+10\theta^4+42\theta^3+84\theta^2+72\theta+24}{\sqrt{2}(\theta^3+6\theta^2+12\theta+6)^{3/2}},$$

$$\beta_2 = \frac{\theta^7+22\theta^6+184\theta^5+780\theta^4+1800\theta^3+2256\theta^2+1440\theta+360}{2(\theta^3+6\theta^2+12\theta+6)^2}$$

We propose another size-biased Poisson-Lindley (SBPL) distribution which is obtained by compounding the size-biased Poisson distribution with Lindley distribution without considering its size-biased form. (This paper is accepted by IJMRS for the process of publication in Dec., 2013 Volume)[12]. The size-biased Poisson distribution has the p.m.f.

$$f(x/\lambda) = \frac{e^{-\lambda}.\lambda^{x-1}}{(x-1)!};\quad x = 1,2,3,\ldots\ldots..;\ \lambda > 0, \ldots\ldots\ldots (1.9)$$

Now if its parameter λ follows the Lindley distribution with p.m.f. (1.1) then the p.m.f. of the size-biased Poisson-Lindley (SBPL) distribution is obtained as:

$$f(x;\theta) = \int_0^\infty \mathrm{f}(x/\lambda)\ .\ \mathrm{g}_0(\lambda;\theta)\, d\lambda$$

$$= \frac{\theta^2}{(1+\theta)^{x+2}}\,.(x+\theta+1), \qquad x = 1,2,3,\ldots\ldots\ldots (1.10),$$

We obtained the first four raw moments and their corresponding central moments of this size-biased Poisson-Lindley (SBPL) distribution (1.10) are as:

Raw Moments:

$$\mu_1' = \frac{\theta^2+2\theta+2}{\theta(\theta+1)}$$

$$\mu_2' = \frac{\theta^3+4\theta^2+8\theta+6}{\theta^2(\theta+1)}$$

$$\mu_3' = \frac{\theta^4+8\theta^3+26\theta^2+42\theta+24}{\theta^3(\theta+1)}$$

$$\mu_4' = \frac{\theta^5+16\theta^4+80\theta^3+210\theta^2+264\theta+120}{\theta^4(\theta+1)}$$

And Central Moments:

$$\mu_1 = \frac{\theta^2 + 2\theta + 2}{\theta(\theta+1)}$$

$$\mu_2 = \frac{\theta^3 + 4\theta^2 + 6\theta + 2}{\theta^2(\theta+1)^2}$$

$$\mu_3 = \frac{\theta^5 + 7\theta^4 + 22\theta^3 + 32\theta^2 + 18\theta + 4}{\theta^3(\theta+1)^3}$$

$$\mu_4 = \frac{\theta^7 + 15\theta^6 + 87\theta^5 + 258\theta^4 + 406\theta^3 + 338\theta^2 + 144\theta + 24}{\theta^4(\theta+1)^4}$$

Thus the mean, variance, skewness, kurtosis and their coefficients are proposed are as:

$$Mean = \frac{\theta^2 + 2\theta + 2}{\theta(\theta+1)},$$

$$Vairance = \frac{\theta^3 + 4\theta^2 + 6\theta + 2}{\theta^2(\theta+1)^2},$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(\theta^5 + 7\theta^4 + 22\theta^3 + 32\theta^2 + 18\theta + 4)^2}{(\theta^3 + 4\theta^2 + 6\theta + 2)^3},$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\theta^7 + 15\theta^6 + 87\theta^5 + 258\theta^4 + 406\theta^3 + 338\theta^2 + 144\theta + 24}{(\theta^3 + 4\theta^2 + 6\theta + 2)^2},$$

$$\gamma_1 = \sqrt{\beta_1} = \frac{\theta^5 + 7\theta^4 + 22\theta^3 + 32\theta^2 + 18\theta + 4}{(\theta^3 + 4\theta^2 + 6\theta + 2)^{3/2}},$$

And

$$\gamma_2 = \beta_2 - 3 = \frac{\theta^7 + 12\theta^6 + 63\theta^5 + 174\theta^4 + 250\theta^3 + 182\theta^2 + 72\theta + 12}{(\theta^3 + 4\theta^2 + 6\theta + 2)^2}$$

II. PROPOSED PSBL DISTRIBUTION

In this paper we propose Poisson size-biased Lindley (PSBL) distribution which is obtained by compounding the Poisson distribution without considering its size-biased form with size-biased Lindley distribution. The Poisson distribution has the p.m.f.

$$g(x/\lambda) = \frac{e^{-\lambda}.\lambda^x}{x!}; \quad x = 0,1,2, \ldots \ldots..; \; \lambda > 0, \ldots \ldots \ldots (2.1)$$

Now if its parameter λ follows the size-biased Lindley (SBL) model with p.d.f. (1.7) then the p.m.f. of the Poisson size-biased Lindley (PSBL) distribution is obtained as:

$$f(x;\theta) = \int_0^\infty \mathrm{f(x/\lambda)}\ .\ \mathrm{h}(\lambda;\theta)\, d\lambda = \int_0^\infty \frac{e^{-\lambda}.\lambda^x}{x!}\ .\ \frac{\theta^3}{(\theta+2)}.\lambda(1+\lambda).e^{-\theta\lambda}\, d\lambda =$$

$$\frac{\theta^3}{\theta+2}\frac{1}{x!}\left[\int_0^\infty e^{-(\theta+1)\lambda}.\lambda^{x+1} d\lambda + \int_0^\infty e^{-(\theta+1)\lambda}.\lambda^{x+2} d\lambda\right]$$

$$= \frac{\theta^3}{\theta+2}.\frac{1}{x!}\left[\frac{1}{(\theta+1)^{x+2}}.x!\,(x+1) + \frac{1}{(\theta+1)^{x+3}}.x!\,(x+1)(x+2)\right]$$

$$= \frac{\theta^3}{(1+\theta)^{x+3}}.\frac{1}{\theta+2}.(x+1).(x+\theta+3), \qquad x = 0,1,2,, \ldots \ldots \ldots (2.2),$$

Here we get, $\sum_{x=1}^{\infty} f(x;\theta) = 1$.

The first four raw moments and their corresponding central moments of this Poisson size-biased Lindley (PSBL) distribution (2.2) comes out to be:

Raw Moments:

$$\mu_1' = E(X) = \frac{2\theta^3 + 10\theta^2 + 14\theta + 6}{\theta(\theta+2)(\theta+1)^2}$$

$$\mu_2' = E(X^2) = \frac{2\theta^4 + 16\theta^3 + 50\theta^2 + 60\theta + 24}{\theta^2(\theta+2)(\theta+1)^2}$$

$$\mu_3' = E(X^3) = \frac{2\theta^5 + 28\theta^4 + 146\theta^3 + 336\theta^2 + 336\theta + 120}{\theta^3(\theta+2)(\theta+1)^2}$$

$$\mu_4' = E(X^4) = \frac{2\theta^6 + 52\theta^5 + 410\theta^4 + 1512\theta^3 + 2712\theta^2 + 2280\theta + 720}{\theta^4(\theta+2)(\theta+1)^2}$$

And Central Moments:

$$\mu_1 = \mu_1' = \frac{2\theta^3 + 10\theta^2 + 14\theta + 6}{\theta(\theta+2)(\theta+1)^2}$$

$$\mu_2 = \frac{2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12}{\{\theta(\theta+2)(\theta+1)^2\}^2}$$

$$\mu_3 = \frac{2\theta^{11} + 32\theta^{10} + 234\theta^9 + 1012\theta^8 + 2842\theta^7 + 5424\theta^6 + 7190\theta^5 + 6644\theta^4 + 4212\theta^3 + 1752\theta^2 + 432\theta + 48}{\{\theta(\theta+2)(\theta+1)^2\}^3}$$

$$\mu_4 = \frac{\begin{array}{c}2\theta^{15} + 60\theta^{14} + 776\theta^{13} + 5848\theta^{12} + 28988\theta^{11} + 100792\theta^{10} + 255416\theta^9 + 482752\theta^8 + 689090\theta^7 + \\ 745548\theta^6 + 608240\theta^5 + 368376\theta^4 + 160656\theta^3 + 47712\theta^2 + 8640\theta + 720\end{array}}{\{\theta(\theta+2)(\theta+1)^2\}^4}$$

Thus the mean, variance, skewness, kurtosis and their coefficients are obtained as:

$$Mean(\mu_1) = \mu_1' = \frac{2\theta^3 + 10\theta^2 + 14\theta + 6}{\theta(\theta+2)(\theta+1)^2},$$

$$Variance(\mu_2) = \frac{2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12}{\{\theta(\theta+2)(\theta+1)^2\}^2},$$

$$\beta_1 = \frac{(2\theta^{11} + 32\theta^{10} + 234\theta^9 + 1012\theta^8 + 2842\theta^7 + 5424\theta^6 + 7190\theta^5 + 6644\theta^4 + 4212\theta^3 + 1752\theta^2 + 432\theta + 48)^2}{(2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12)^3},$$

$$\beta_2 = \frac{\begin{array}{c}2\theta^{15} + 60\theta^{14} + 776\theta^{13} + 5848\theta^{12} + 28988\theta^{11} + 100792\theta^{10} + 255416\theta^9 + 482752\theta^8 + 689090\theta^7 + \\ 745548\theta^6 + 608240\theta^5 + 368376\theta^4 + 160656\theta^3 + 47712\theta^2 + 8640\theta + 720\end{array}}{(2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12)^2},$$

$$\gamma_1 = \frac{2\theta^{11} + 32\theta^{10} + 234\theta^9 + 1012\theta^8 + 2842\theta^7 + 5424\theta^6 + 7190\theta^5 + 6644\theta^4 + 4212\theta^3 + 1752\theta^2 + 432\theta + 48}{(2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12)^{3/2}},$$

And

$$\gamma_2 = \frac{\begin{array}{c}2\theta^{15} + 48\theta^{14} + 860\theta^{13} + 12500\theta^{12} + 48728\theta^{11} + 176956\theta^{10} + 566162\theta^9 + 2160028\theta^8 + 882278\theta^7 + \\ 763716\theta^6 + 492536\theta^5 + 262695\theta^4 + 107032\theta^3 + 36480\theta^2 + 3456\theta + 288\end{array}}{(2\theta^7 + 20\theta^6 + 84\theta^5 + 188\theta^4 + 242\theta^3 + 180\theta^2 + 72\theta + 12)^2}$$

We now give some basic properties of the SBPL model.

(i) Since

$$\mu - \sigma^2 = -\frac{2\theta^6 + 20\theta^5 + 72\theta^4 + 128\theta^3 + 122\theta^2 + 60\theta + 12}{\{\theta(\theta+2)(\theta+1)^2\}^2},$$

It follows that $\mu <(=)(>)\ \sigma^2$ for $<(=)(>)\ \theta^*$, where $\theta^* \cong 1.671162$. That is the SBPL distribution is over-dispersed (equi-dispered) (under-diapered) for $\theta <(=)(>)\ \theta^*$.

(ii) Since

$$\frac{f(x+1;\theta)}{f(x;\theta)} = \left(\frac{1}{\theta+1}\right).\left(1+\frac{1}{x+1}\right).(1+\frac{1}{x+\theta+3})$$

Is a decreasing function in x, f(x; θ) is log-concave. Therefore the PSBL distribution is unimodal, has an increasing failure rate (IFR) (and hence, increasing failure rate average (IFRA), new better than used (NBU), new better than used in expectation (NBUE) and decreasing mean residual life (DMRL) in Barlow and Proschan (1981) for more details about the definition of these aging concepts are given.

III. METHOD OF MOMENTS

Given a random sample $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$, of size n from the SBPL distribution with p.m.f.(2.2), the MOM estimate, $\tilde{\theta}$ of θ is given by

$$E(X) = \frac{2\theta^3 + 10\theta^2 + 14\theta + 6}{\theta(\theta+2)(\theta+1)^2} = \bar{x}$$

Or, $2\theta^3 + 10\theta^2 + 14\theta + 6 = \theta(\theta+2)(\theta+1)^2 \times \bar{x}$
Or, $2\theta^3 + 10\theta^2 + 14\theta + 6 = (\theta^4 + 4\theta^3 + 5\theta^2 + 2\theta).\bar{x}$
Or, $\theta^4 + (2\bar{x}-1)2\theta^3 + (\bar{x}-2)5\theta^2 + (\bar{x}-7)2\theta - 6 = 0$

Note that $\bar{x}$ = 1 if and only if x_i = 1 for all i = 1, 2,…………..,n. A data set where all observations are ones is not worth analyzing. This situation, of course, will not lead to any estimate of θ. However, such situation may arise in a simulation experiment when n is small. For this reason, we will assume throughout this paper that $\bar{x} > 1$.

IV. MAXIMUM LIKELIHOOD ESTIMATION

Given a random sample $x_1, x_2, \ldots, x_n$, of size n from the SBPL distribution with p.m.f. (2.2) is,

$$f(x;\theta) == \frac{\theta^3}{(1+\theta)^{x+3}}.\frac{1}{\theta+2}.(x+1).(x+\theta+3)$$

The likelihood function will be:

$$L(x_{i;}\theta) = \prod_{i=1}^{n} f(x_{i;}\theta)$$

$$= \left(\frac{\theta^3}{(\theta+1)^3}.\frac{1}{\theta+2}\right)^n.\prod_{i=1}^{n}\frac{1}{(\theta+1)^{x_i}}.(x_i+1)(x_i+\theta+3)$$

$$logL = 3n\, log\theta - 3n\, log(\theta+1) - n\, log(\theta+2)$$

$$+\sum_{i=1}^{n}\log(x_i+1)+\sum_{i=1}^{n}log(x_i+\theta+3)$$

$$\therefore \frac{\partial logL}{\partial\theta} = \frac{3n}{\theta} - \frac{3n}{\theta+1} - \frac{n}{\theta+2} + \sum_{i=1}^{n}\frac{1}{x_i+1}\sum_{i=1}^{n}\frac{1}{x_i+\theta+3}$$

And

$$\therefore \frac{\partial^2 logL}{\partial\theta^2} = -\frac{3n}{\theta^2} + \frac{3n}{(\theta+1)^2} + \frac{n}{(\theta+2)^2} - \sum_{i=1}^{n}\frac{1}{(x_i+1)^2} - \sum_{i=1}^{n}\frac{1}{(x_i+\theta+3)^2}$$

Thus the ML estimate $\hat{\theta}$ of θ is the solution of the non-linear equation:

$$\frac{3n}{\theta} - \frac{3n}{\theta+1} - \frac{n}{\theta+2} + \sum_{i=1}^{n}\frac{1}{x_i+1}\sum_{i=1}^{n}\frac{1}{x_i+\theta+3} = 0 \ldots\ldots\ldots\ldots (4.1)$$

The solution may obtained by appropriate numerical methods.

V. SIMULATION STUDY

A simulation may be done using an algorithm to generate random samples from this Poisson-size biased Lindley distribution, a simulation study may be carried out N = 10,000 times for each pair (θ, n) where θ = 0.5,1, 2, 8 and n = 20 (20) 100. The study calculates the following measures:

(i) Average bias of the simulated estimates:

$$\frac{1}{N}\sum_{i=1}^{N}(\theta_i^* - \theta)$$

Where θ_i^* is the MOM estimate $\tilde{\theta}_i$ or the ML estimate $\hat{\theta}_i$.

(ii) Average mean-square error (MSE) of the simulated estimates:

$$\frac{1}{N}\sum_{i=1}^{N}(\theta_i^* - \theta)^2.$$

(iii) Coverage probability = percentage of confidence intervals containing θ.

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